A
PLAN
OF A
COURSE OF LECTURES
ON THE
PRINCIPLES
OF
NATURAL PHILOSOPHY.

BY THE

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THE following Plan of a Courſe of Lectures on the Principles of Natural Philoſophy was drawn up in conſequence of the author's appointment to read the *Plumian Lectures* on *Experimental Philoſophy* and *Practical Aſtronomy*. Some propoſitions are added, beſides thoſe which can be ſubjected to experimental proof, in order to preſerve their connection, that it might at the ſame time direct the theoretical ſtudent. Such obſervations are added as were judged might be uſeful, both to illuſtrate the experiments and aſſiſt the reader.

MECHANICS.

ON ATTRACTION AND REPULSION.

Pr. 1. ALL bodies upon the earth's ſurface have a tendency to move towards it.

This tendency of bodies towards the earth ariſes from the compound tendency towards all its parts. For by an experiment made by Dr. MASKELYNE upon the ſide of the mountain Schehallian, he found that a pendulum had a tendency towards the mountain. Allowing therefore the tendency which all bodies upon the earth have to every part of the earth, it will follow that every part of the earth has a like tendency to every body upon the earth. For if *A* have a tendency to *B*, and *A* and *B* be matter of the ſame kind, *B* muſt have a like tendency to *A*. The tendency of the pendulum towards the mountain was obſerved to be much leſs than the tendency of a body towards the whole earth. The tendency therefore of *A* towards *B* appears to be greater the greater *B* is. A ſmall body will therefore have a greater tendency towards a large one, than a large body has towards a ſmall one. This power by which bodies are thus made to approach each other is called attraction of GRAVITATION.

If a glaſs bubble be placed upon water in a veſſel near to its ſide, it is obſerved to move up to the ſide. Or if two light bodies be laid upon water near to each other, they will move up to each other, and the ſmalleſt body will move the faſteſt. Theſe have been brought as inſtances of the attraction of the bubble to the ſide of the veſſel, and of the mutual attraction of the bodies towards each other; but the motions which they are obſerved to have are much greater than what could ariſe from the attraction of gravitation. If a cork be laid upon water, and another cork be holden near to it without touching the water, it will produce no effect; but as ſoon as the cork is put into the water the other cork moves up to it. They do not come together therefore by their mutual attraction.

2. If two glaſs planes be moiſtened with oil and inclined at a very ſmall angle, and a drop of oil be put between them, it will move towards their concourſe.

This proves that there is an attraction between the glaſs and the oil. The attraction which is obſerved to take place between two bodies in contact, is called the attraction of COHESION.

3. If two globules of quickſilver lying on a ſmooth plane be brought to touch, they immediately ruſh together and form one globule.

This can ariſe only from their ſtrong attraction. Drops of water will do the ſame.

4. Water is attracted by glaſs.

This appears from its riſing at the edge of a glaſs veſſel above the level of the other part. Alſo if very ſmall glaſs tubes be dipt in water, the water will ſtand in them above the level of the water in the veſſel. Or if two glaſs planes forming a ſmall angle be immerſed perpendicularly in water, the water riſes between them above the ſurface in the veſſel, and the height is greater the nearer to the concourſe of the planes. The curve terminating the top of the water between the planes is an hyperbola.

5. Quickſilver is attracted by glaſs.

For a globule of quickſilver will adhere to the under ſide of a clean piece of glaſs. Its attraction therefore to the glaſs muſt be greater than its gravity.

6. The particles of water are more ſtrongly attracted by glaſs than by each other.

This appears from the riſing of the water in ſmall glaſs tubes, and between the glaſs planes, above the level of the water in the veſſel.

7. The particles of quickſilver attract each other more than they are attracted by glaſs.

For if a glaſs tube be put into quickſilver, the quickſilver will not riſe in it ſo high as the ſurface of the quickſilver in the veſſel. Alſo if two glaſs planes be put perpendicularly into quickſilver, and be inclined at a ſmall angle, the quickſilver will ſtand at a leſs height between them than that of the ſurface of the fluid in the veſſel. The curve terminating the ſurface is an hyperbola.

8. Mercury

8. Mercury attracts ſilver more ſtrongly than it does lead, lead more ſtrongly than braſs and braſs more ſtrongly than ſteel.

It is remarkable that the chemical affinities of theſe bodies to mercury follow the ſame order.

9. If two poliſhed plane ſurfaces of metal, &c. be beſmeared with oil, greaſe, &c. and preſſed together, they are obſerved to cohere very ſtrongly.

The oil fills up the cavities and diſlodges the air, which prevents the attraction of coheſion from taking place. If the air be expelled by different ſubſtances, as oil, turpentine, greaſe, &c. it is found that the attraction of coheſion is different. Theſe ſubſtances therefore tend, in ſome way or other, to affect the attraction. It is this attraction of coheſion by which the parts of a body are kept together. When you break a body you only overcome this attraction, and could you join again the parts exactly in the ſame manner, it would be as ſtrong as it was before. The ſoldering of metals, glueing of bodies, &c. is only the bringing of the conſtituent particles ſo near that the attraction of coheſion may take place. The ſolder and the glue attach themſelves to each body upon this principle, and thus connect the bodies. It ſeems as if a contact of the parts muſt take place before this effect can be produced, for if the body be not well cleaned the ſolder will not hold them ſo firmly together.

Hence we may explain why ſome bodies are hard, others ſoft, others fluid, &c. Hard bodies may conſiſt of particles which touch in a great part of their ſurfaces, and thus they will have a great power of attraction. The conſtituent particles of ſoft bodies may touch in a leſs quantity of ſurface, by which the attraction will be weakened. And fluids may conſiſt of globular particles, which touching each other only in one point, their attraction muſt be ſo weak as to permit them to move with the utmoſt facility amongſt each other. Elaſticity may ariſe from the particles of a body when diſturbed not being drawn out of each others attraction; as ſoon therefore as the force upon it ceaſes to act, they reſtore themſelves to their former poſition.

Solids are diſſolved in menſtruums from the particles of the ſolid being more attracted to the fluid than to themſelves. And precipitation ariſes from the ſame cauſe; for if to the ſolution of any ſolid in a fluid, ſome other ſolid or fluid be added whoſe particles are attracted by the fluid with a greater force than thoſe of the ſolid which was diſſolved, that ſolid becomes diſengaged and falls to the bottom in a fine powder. Thus ſilver diſſolved in aqua fortis is precipitated by copper.

10. If ſeveral bodies, as glaſs, amber, ſealing

 wax, &c.

wax, &c. be rubbed with dry woollen cloth, they will both attract and repel light bodies.

This is called ELECTRICAL attraction and repulsion.

11. The same poles of two magnets repel, and the contrary poles attract each other.

This is called MAGNETIC attraction and repulsion.

The existence of an elastic fluid proves that the particles must be kept at a distance by a repulsive force. The difficulty of mixing many fluids, as oil and water, probably arises from the repulsion between the particles. That metals when dissolved in fluids should diffuse themselves equally through the fluid, has been supposed to be owing to a repulsive force, which takes place after the particles are separated to a certain distance.

From the principle of attraction and repulsion Sir I. NEWTON accounts for all the phænomena of nature. By the attraction of gravitation he accounts for the motions of the heavenly bodies; and for the motions between the component particles of bodies by attraction and repulsion. But why the particles should in some cases attract and in others repel, he attempts not to explain. See his Optics, Qu. 31.

LAWS OF MOTION.

12. Every body perseveres in its state of rest or uniform motion in a right line, until some external force acts upon it.

For no effect can be produced without a cause, and no cause is here supposed to act, the body being supposed to be void of self motion. The force with which a body resists any change, is called its VIS INERTIÆ.

13. The change of motion is in proportion to the force impressed, and takes place in the direction in which the force acts.

The first part of this is only measuring the effect by the cause, supposed to act for the same time; and the second part is manifest, it being evident that the effect of the force must take place in the line in which it acts.

14. Action and reaction are equal and contrary.

The meaning of this is, that when two bodies move in opposite directions and strike each other, they lose equal quantities of motion in their

their respective directions, measuring the motion by the velocity and quantity of matter conjointly; and if they move in the same direction, the quantity of motion which the striking body loses in that direction, the other gains. Hence the quantity of motion in the same direction is not altered from the collision of bodies.

This may be proved either directly by experiments, or by assuming the principle and showing that the theory of the collision of bodies agrees in all cases with the experiments. Sir. I. NEWTON established its truth by experiments of the latter kind, by suspending two elastic balls and letting one descend in a circular arc and strike the other; and by estimating the velocity of the striking body before impact and of each after, he found the law to obtain. See the PRINCIPIA, Scholium to the Laws of Motion.

These three laws of motion are assumed by Sir I. NEWTON as the fundamental principles of mechanics; and the theory of all motions deduced from them, as principles, being found to agree in all cases with experiments and observations where they can be applied, these laws are considered as mathematically true.

ON THE COMPOSITION AND RESOLUTION OF MOTION, AND THE MECHANICAL POWERS.

15. If a body be in motion and another body be projected from it, the latter body, besides its projectile motion, will retain the motion of the former body.

It was at first objected to the earth's motion, that in that case a stone thrown perpendicularly upwards ought not to fall down again in the same place, but to be left behind. But the contrary admits of a very satisfactory proof by experiment.

16. If a body be acted upon by two single impulses, or by any two continued forces each of which always acts parallel to itself, it will not by one of the motions A be hindered by the other B from approaching a line parallel to the direction in which B takes place.

Upon this principle depend the composition of motion and the doctrine of projectiles.

17. If a body be acted upon by any two single im-

impulſes, it will deſcribe the diagonal of a parallelogram in the ſame time it would have deſcribed either ſide, had the forces acted ſeparately.

18. If a body be kept at reſt by three forces, and in the directions in which they act lines be drawn from the body proportional to them, and any two of theſe lines be completed into a parallelogram, the diagonal will be equal and oppoſite to the third line.

Hence any two forces may be compounded into one which ſhall in every reſpect be equivalent to them; and therefore converſely, any ſingle force may be reſolved into two in any two directions, by deſcribing upon the line repreſenting the ſingle force a parallelogram whoſe ſides ſhall lie in the required directions.

19. When a body is kept at reſt by three forces, they will be as the three ſides of a triangle parallel to the directions in which they act.

For they are repreſented by the diagonal and two ſides of the parallelogram forming a triangle, two of whoſe ſides lie in the direction of two of the forces, and the third ſide parallel to the third force, and if lines be drawn parallel to theſe directions they will form a ſimilar triangle, and therefore the proportion of the ſides will be the ſame. It follows alſo that the three forces will be as the reſpective ſides of a triangle perpendicular to which they act, becauſe ſuch a triangle will be ſimilar to the other. The three forces muſt be all directed to the ſame point, otherwiſe they will give the body a rotatory motion.

Cor. Hence the converſe is true, that if a body be acted upon by three forces proportional to the three ſides of a triangle parallel to which they act, it will be at reſt.

20. If a body B reſt upon a ſtring, between two pullies in an horizontal poſiton, being balanced by two equal bodies A, A hanging on oppoſite ſides, the diſtance of B from the horizontal line joining the pullies will be $\frac{B\,d}{\sqrt{4\,A^2 - B^2}}$, d being half the diſtance of the pullies.

If

If $d = 12$ in. $A = 4$ oz. $B = 3$ oz. the diſtance of B from the horizontal line $= 4{,}85$ in.,

21. If the power and reſiſtance act perpendicular to the ſides of a wedge at reſt, they will be as the three reſpective ſides.

This follows from the obſervation to propoſition 19, by ſubſtituting a triangle for the body, and conceiving the three forces to act perpendicular to the three ſides.

By the reſolution of motion, the effect of any force oblique to a plane in a direction perpendicular to it, varies as the force × ſin. Incl. therefore if the power P and reſiſtances R, R', act obliquely to the ſides of the wedge at the angles p, r, r' reſpectively, the back and reſpective ſides will be as $P \times$ ſin. p, $R \times$ ſin. r and $R' \times$ ſin. r'. It is here ſuppoſed that the part of the force acting parallel to the ſide is all loſt on account of the obliquity at which the forces act, which can only hold upon ſuppoſition that there is no friction.

When a wedge is driven into a piece of wood, the friction is greater than the power neceſſary to preſerve the equilibrium; for when the power ceaſes to act at the back, the friction prevents the reſiſtances from driving it out, which they neceſſarily would when the power was removed, were it not for the friction. In like manner nails, pegs, &c. are retained by friction, for being made tapering, the effect of the reſiſtance of the wood is to drive them out.

In the eſtimation of the proportion of the power to the weight in all the mechanic powers, there is ſuppoſed to be an equilibrium between them.

22. If any two weights balance each other when hung upon a ſtraight lever, they will be to each other inverſely as their diſtances from the fulcrum.

Hence when the diſtances are equal the weights are equal, which is the caſe with the common ſcales.

If the arms a, b, are not of an equal length, the true weight v will be a geometrical mean between the weights m and n which will balance it when hung firſt on one end and then on the other; for let $\left.\begin{array}{l} v : m :: a : b \\ v : n :: b : a \end{array}\right\}$ $\therefore v^2 : mn :: ab : ab$, hence $v^2 = mn$ and $m : v :: v : n$. Now as a geometrical mean is greater than an arithmetical, half $m + n$ is greater than v, conſequently half the ſum of the two weights on a falſe balance gives more than the true weight.

It follows alſo from this prop. that when two men carry a weight upon a lever lying on their ſhoulders, that the part which each bears is inverſely as his diſtance from the weight.

A lever is defined to be an inflexible rod void of gravity and moveable about a fulcrum. But as every body has gravity, the lever is balanced

lanced before the weights are applied, ſo that no effect but that of the weights are to be conſidered. A ſtraight lever is that where the points to which the weights are applied and the fulcrum are in the ſame ſtraight line, in which caſe, if the lever be in equilibrio in one poſition, it will in all others, and therefore its weight may in all caſes be neglected. A bent lever muſt alſo be ſo conſtructed as to be in equilibrio in any poſition, otherwiſe its property cannot be experimentally proved; that is, by conſtructing the lever ſo that the center of ſuſpenſion may be the center of gravity. A ſtraight lever having the forces applied obliquely becomes ſimiliar to a bent lever, ſo far as the forces produce an equilibrium; for after reſolving each force into two, one in the direction of the arm and the other perpendicular to it, the effect of the latter upon the arms to balance each other will be the ſame, whether the arms lie in the ſame direction from the fulcrum, or form there an angle.

Hence it alſo appears, that the effect of a weight upon a lever to turn it about, is as the weight multiplied into its diſtance from the fulcrum; for the effects are always equal when theſe products are equal, and therefore the effects muſt be meaſured by the products.

Two equal weights hanging at unequal diſtances from the fulcrum will balance, if the velocities with which they move, when put in motion, be equal. This is called the MECHANICAL PARADOX.

23. If the weights act obliquely on the arms of a bent lever, they will be inverſely as the perpendiculars from the fulcrum to the lines of direction.

When the fulcrum is between the power and weight, the lever is called of the FIRST kind; if the weight be between the fulcrum and power it is ſaid to be of the SECOND kind; and if the power be between the weight and fulcrum it is called of the THIRD kind.

24. In a compound lever, the power : the weight :: product of the lengths of the arms of the levers lying on the contrary ſides of the fulcrums to the power : product of the lengths of the other arms.

25. In the wheel and axle, the power : the weight :: radius of the axle : radius of the wheel.

Cor. Hence if there be ſeveral wheels ſo conſtructed, that the periphery of one may act upon the axle of the other, the power : the weight :: the product of the radii of all the axles : the product of the radii of all the wheels.

26. On the inclined plane, the power : the weight

weight :: the ſine of the plane's inclination : the coſine of the angle which the ſtring going from the weight makes with the plane; and the weight : the preſſure :: the coſine of the ſame angle : the ſine of the angle between the directions of the power and weight.

Cor. 1. Hence the power to ſuſtain a given weight on a given plane is leaſt, when the ſtring from the weight is parallel to the plane, and their ratio is then that of the height : the length of the plane; and the weight : the preſſure :: the length : the baſe.

Cor. 2. When the ſtring from the weight is parallel to the height, the power is equal to the weight, and the preſſure vaniſhes.

Cor. 3. When the ſtring is parallel to the baſe, the power, weight and preſſure are as the height, baſe and length.

The preſſure of the weight is here greater than the weight itſelf, becauſe the power preſſes the weight upon the plane.

27. In a fixed pulley, the power is equal to the weight.

Although there is here no mechanical advantage, there is frequently a conveniency, by altering the direction in which the power is applied.

28. If the ſame ſtring go round two ſets of pullies in two blocks, the power : the weight :: unity : the number of ſtrings at the lower block.

In this caſe the lower block cooperates with the weight, by being raiſed with it.

29. If each pulley have a ſeparate ſtring fixed to ſomething immoveable above, the power : the weight :: unity : that power of 2 whoſe index is the number of moveable pullies.

Here all the moveable pullies cooperate with the weight, as they riſe with it.

30. If each pulley have a ſeparate ſtring fixed to the weight, the power: the weight :: $1 : 2^n - 1$, where n is the whole number of the pullies.

In this set of pullies, the moveable pullies cooperate with the power by descending with it.

Hence in all the cases of the pullies, the pullies themselves must first be balanced, and then the proportion of the power to the weight will be as given in the propositions.

31. In the screw, the power : the weight :: the distance of two contiguous threads in a direction parallel to the axis of the screw : the circumference described by the power.

In the common screws the friction is generally equal to the power at least, for when the power is not applied the weight does not make the screw run down.

In all the above propositions respecting the equilibrium of the power and weight, there has been supposed to be an equilibrium between the parts of the machine before the power and weight were applied, and no friction to hinder either of them from giving motion to it, so that when both are applied, the least power added to either will destroy the equilibrium. In cases therefore where the friction is considerable, and its effect cannot be estimated, no experimental proof of the proposition can be applied. In machines made of wood, the friction is generally estimated at one third of the whole weight, that is, that if a power equal to one third of the weight applied to the machine could be applied to the weight without adding any more friction, it would overcome the friction; but as, by adding this power, more friction is added to the machine, therefore one third of that power applied must be again added to overcome the friction occasioned by that power, and so on ad infinitum; hence if $W =$ the whole weight on the machine, $\frac{1}{3} W + \frac{1}{9} W$ + &c. ad inf. $= \frac{1}{2} W$ the power to be applied to overcome the resistance. Or if instead of the friction being $\frac{1}{3} W$ if we suppose it $\frac{1}{n} W$, then $\frac{1}{n-1} W$ is the power to be applied to overcome it.

32. In a machine compounded of any number of mechanic powers, the power is to the weight as the sum of the ratios expressing the ratio of the power to the weight in each.

For example, if a power and weight act upon the arms of a lever, and the weight lie upon an inclined plane to which the string is parallel, then the power is to the weight in a ratio compounded of the in-

verse

verſe ratio of the perpendiculars from the fulcrum upon the lines of direction, and of the length of the plane to the height.

In every machine, by diminiſhing the power we increaſe the time, and in the ſame proportion, becauſe the velocity of the power : the velocity of the weight (the velocities being eſtimated in the directions in which they oppoſe each other) :: the weight : the power. In the theory of this ſcience we ſuppoſe all planes and bodies perfectly ſmooth, levers to have no weight, chords to be perfectly pliable, and machines to have no friction. The allowances to be made for the difference beween theory and practice from theſe circumſtances muſt be determined by experiment.

ON THE CENTER OF GRAVITY.

Def. The center of gravity is that point in a body or ſyſtem of bodies, by which, if it were ſuſpended, it would reſt in any poſition.

33. If a ſtraight rod be balanced upon a fulcrum, and weights be hung upon each ſide, there will be an equilibrium when the ſums of the products of each weight multiplied into its diſtance from the fulcrum on each ſide are equal.

This appears from obſervation the 5th. to prop. 22.

34. If any point be aſſumed in the rod, the diſtance of the center of gravity from that point will be equal to the ſum of the products of each body multiplied into its diſtance from that point, (thoſe products being reckoned negative when the bodies lie on the contrary ſide of that point to the center of gravity,) divided by the ſum of the bodies.

35. The effect of any number of weights applied to a lever to turn it about any point, is juſt the ſame as if all the weights were collected into their common center of gravity.

For by the laſt obſervation it appears, that the ſum of the products of each body multiplied into its diſtance from any point, is equal to the ſum of all the bodies multiplied into the diſtance of their center of gravity from that point, and the effect to turn a lever about any point is meaſured by the body into its diſtance from that point; therefore con-

ſidering all the bodies as placed in their center of gravity the effect muſt continue the ſame.

36. Any three bodies connected together have a center of gravity.

For the effect of any two is the ſame as if they were placed in their center of gravity, by the laſt; hence, conceive them to be placed there, and the center of gravity of that and the third body, is the center of gravity of the three.

Hence therefore any ſyſtem of bodies, however ſituated, has a center of gravity.

It follows alſo from the ſame principle that every body has a center of gravity; for every body may be conceived to be reſolved into an infinite number of corpuſcles, any number of which, however ſituated, have a center of gravity.

Hence as the effect of the whole body is the ſame as if all the matter were collected into the center of gravity, we may conceive it to be all concentrated into that point, and the effect to produce an equilibrium, or to generate motion on a lever, will remain the ſame. Hence the PLACE of a body is underſtood to be that point where the center of gravity is.

But although the effect of a body to produce motion may be the ſame as when all the matter is conceived to be concentrated in its center of gravity, yet the effect produced will not be the ſame, owing to the inertiæ being different. In the doctrine of equilibrium we have only weight to conſider, whereas in the doctrine of motion we have to conſider both the power which gives motion and the reſiſtance of the body from its inertia to oppoſe the communication of that motion.

37. If a body be ſuſpended by a ſtring and drawn from its perpendicular poſition, it will be accelerated by a force which is as the ſine of its angular diſtance from the loweſt point.

Hence the place of a body being denoted by its center of gravity, it follows that when any body is ſuſpended, it cannot reſt till its center of gravity comes to the loweſt point, becauſe in any other ſituation it will be acted upon by an accelerating force. Hence if a body, or ſyſtem of bodies, be not ſuſpended by a ſtring, but by a fulcrum or axis paſſing through ſome point in it which is not the center of gravity, it will reſt when the center of gravity is either directly above or below the point of ſuſpenſion.

It is upon this principle that a double cone appears to roll up two inclined planes forming an angle with each other and lying in the ſame plane; for as it rolls up it ſinks between them, and by that means the center of gravity actually keeps deſcending. To effect this the height of the planes muſt be LESS than the radius of the baſe of the cone:

cone: if the height be EQUAL to the radius, the body will reſt in any part of it; and if the height be GREATER than the radius, it will deſcend. A cylinder may alſo roll up an inclined plane for a ſmall diſtance, if it be loaded on one ſide with ſomething heavier than itſelf, and that ſide be laid towards the top of the plane, for then the center of gravity being out of the axis towards that part, it will deſcend whilſt the body rolls upwards.

Upon the ſame principle a body which would fall off a table, will not fall off although you hang a weight upon the part which does not reſt upon the table, provided you, by that means, throw the center of gravity of the whole under the table.

Hence alſo you may eaſily balance a body reſting upon a point on its under ſide, by hanging on a body at each end; for by that means you throw the center of gravity below the point of ſuſpenſion, and then it brings itſelf to its loweſt point, where it reſts. Whereas before the bodies were hung on, the center of gravity was above the point of ſuſpenſion, and unleſs it had been exactly over it, which it is almoſt impoſſible to accompliſh, it would deſcend, and the body muſt fall.

Hence we have a very eaſy practical method of finding the center of gravity of any irregular plane figure. Suſpend it by any point with the plane perpendicular to the horizon, and from the point of ſuſpenſion hang a body ſuſpended by a ſtring, and draw a line upon the body where the ſtring paſſes over; do the ſame for any other point of ſuſpenſion, and where the two lines meet is the center of gravity. For the center of gravity being in each line, it muſt be at the point where they interſect.

The direction of the line by which a body at reſt is ſuſpended is the direction in whieh gravity acts, and if a line be ſo drawn from the center of gravity of a body, it is commonly called the LINE OF DIRECTION.

38. If a line be drawn from any angle of a triangle to biſect the oppoſite ſide, the center of gravity of the triangle will be upon that line and at the diſtance of two-thirds of it from the angle.

Hence we may find the center of gravity of any rectilinear figure: divide it into triangles and find the center of gravity each, and in each center of gravity conceive bodies to be placed equal in weight to its reſpective triangle, or weights proportional to the reſpective areas, and then find the center of gravity of all the bodies.

39. The center of gravity of a parabola lies in its axis at the diſtance of three-fourths of it from the vertex.

The center of gravity of bodies in general can be found only by a

fluxional calculation. In all regular bodies it is manifeſtly in the point which we commonly call the middle. In a cone it is in the axis at the diſtance of three-fourths of it from the vertex. In an irregular body it may happen that the center of gravity may not fall within the body.

40. If there be any number of bodies and perpendiculars be drawn from them to any plane, the diſtance of the center of gravity of all the bodies from that plane, is equal to the ſum of the products of each body multiplied into its perpendicular diſtance from the plane divided by the ſum of the bodies.

Hence by aſſuming three planes and finding the diſtance of the center of gravity from each, you determine the center of gravity of the bodies.

Hence alſo the ſum of the motions of any number of bodies reduced to the ſame direction, will be equal to the motion of all the bodies placed in their center of gravity in the ſame direction.

41. If a circle be deſcribed about the center of gravity of any number of bodies reduced to that circle by lines drawn perpendicular to it, then the ſum of the products of each body into the ſquare of its diſtance from any point of the periphery is the ſame.

42. If two bodies move uniformly in two ſtraight lines, their center of gravity will either be at reſt or move uniformly in a ſtraight line.

43. If a body equal to the ſum of any two bodies be placed in their center of gravity, and the ſame quantities of motion in the ſame directions be communicated to it which are communicated to the two bodies, this body will move in the ſame line which the center of gravity of the two bodies deſcribes, and with the ſame velocity.

Cor. 1. Hence equal and contrary motions communicated to any ſyſtem

ſyſtem of bodies will have no effect upon their center of gravity, for they would not diſturb a body equal to the ſum of them all placed in their center of gravity. For the ſame reaſon the motion of the center of gravity of any number of bodies will not be diſturbed by their colliſion.

Cor. 2. Hence alſo the center of gravity of a ſyſtem of bodies will not be diſturbed by their mutual attractions, as the motions thus communicated are always equal and oppoſite. Hence the center of gravity of our ſyſtem of planets is either at reſt or moves on uniformly in a ſtraight line. The latter is ſuppoſed by Dr. HERSCHEL to be the caſe, from the change which has been obſerved in the relative ſituation of ſome of the fixed ſtars.

44. If one body be at reſt and another deſcribe any curve, the center of gravity will deſcribe a ſimilar curve.

45. If a body be placed upon an horizontal plane, and the line of direction paſs within the baſe, it will ſtand; if it paſs without the baſe, it will fall.

This is manifeſt from the 1ſt. obſervation to prop. 37. for conceiving the baſe to be the fulcrum, the center of gravity is directly over it in one caſe but not in the other.

Our own motions are ſubject to this rule, which we obſerve without thinking of it. When a man ſtands upright, his center of gravity falls between his feet and he is ſupported; but if he lean forward, he throws the line of direction without his baſe, and he would fall if he did not put forward one of his feet ſo as to cauſe it to fall within. For this reaſon, a porter with a load on his back leans forward that the load may not throw the line of direction out of his baſe behind.

Upon this principle alſo it is that a body juſt hung upon the edge of a table will not fall off, becauſe part of the body hanging under the table, the center of gravity of the whole is ſupported.

46. If a body be ſet upon an inclined plane, the line of direction will fall oblique to the baſe; in this caſe, if the line of direction fall within the baſe the body will ſlide down the plane without tumbling; but if it fall without the baſe, the body will roll, or partly ſlide and partly roll, according as the quantity of friction is greater or leſs. If there

there were no friction, the body, of whatever form, would ſlide without rolling.

Hence a globe on a perfectly ſmooth inclined plane would ſlide without rolling; for the force of gravity can give the body no rotatory motion, and as there is no friction there is no force which acts out of the center of gravity to give it a rotation.

ON THE COLLISION OF BODIES.

47. If two nonelaſtic bodies A and B move in the ſame ſtraight line with velocities x and y, and ſtrike each other, their common velocity after impact will be $\frac{Ax \pm By}{A+B}$, where the ſign $+$ muſt be taken when the bodies move in the ſame direction, and — when in oppoſite directions.

Cor. If A ſtrike B at reſt, the velocity after impact $= \frac{Ax}{A+B}$.

This, and the propoſitions reſpecting elaſtic bodies, depend upon the third law of motion, that the quantity of motion in the ſame direction is not altered by the action of two bodies on each other. When two nonelaſtic bodies meet, they act upon each other till they have acquired a common velocity, and then they move on together. Moreover, when two bodies act thus directly upon each other, their inertiæ muſt be ſimply as their velocities and quantities of matter, for the endeavour of each body to oppoſe the communication of motion muſt be ſimply as its reſpective motion, becauſe theſe motions act at no mechanical advantage or diſadvantage.

48. If A and B be two perfectly elaſtic bodies moving in the ſame direction with the velocities x and y, and A ſtrike B, then A's velocity after impact will be $\frac{Ax - Bx + 2By}{A+B}$, and B's will be $\frac{2Ax - Ay + By}{A+B}$. If they move in oppoſite directions,

tions, A's velocity will be $\frac{Ax - Bx - 2By}{A + B}$, and B's will be $\frac{2Ax + Ay - By}{A + B}$.

When two perfectly elastic bodies meet, they first act upon each other till they have acquired a common velocity, as in nonelastic bodies, and then by the endeavour of each body to recover its figure they separate; and when the force with which they separate is equal to the force with which they were compressed together, they are said to be perfectly elastic. The time in which the action between the bodies takes place is of no consequence; also the effect will be the same if only one body be elastic, for in that case they will first act upon each other till they have acquired a common velocity, as in nonelastic bodies, and then the force with which the elastic body endeavours to restore its figure will double the action between them, and therefore if the effect of the compression be the same, the effect of their separation must also be the same, and consequently the whole effect will be the same. If the force with which they separate be less than the force with which they are compressed together, they are said to be imperfectly elastic.

Cor. 1. If A strike B at rest, or $y = o$, A's velocity after impact $= \frac{Ax - Bx}{A + B}$, and B's $= \frac{2Ax}{A + B}$. Hence if $A = B$, A will be at rest after impact, and B will move with the velocity which A had before impact. If B be greater than A, A's velocity becomes negative, and therefore A will be reflected back. If B be less than A, A's velocity will be positive, and therefore A will procede on in the same direction after impact.

Hence if there be any number of bodies of equal magnitude lying in the same straight line, and the first strike the second, all the bodies will rest except the last, which will move off with the velocity of the first before impact. This theory will be found to agree very nearly with experiments made with ivory balls, which are nearly perfectly elastic. If the bodies increase in magnitude, each will be reflected back, and if they decrease, each will go forward after the stroke.

Cor. 2. If $A = B$ and they move in the same direction, A's velocity after impact $= y$, and B's velocity $= x$, hence the bodies have exchanged velocities and continue to move in the same direction. If they move in opposite directions each will be reflected back, having exchanged velocities.

Cor. 3. If A strike an immoveable object, or if B be infinite and $y = o$, A's velocity after impact $= -x$, or A will be reflected back with the same velocity.

Cor. 4. The velocity of A minus that of B before impact $=$ the velocity of B minus that of A after; for B's velocity $-$ A's after impact, when they move in the same direction, is $\frac{2Ax - Ay + By - Ax + Bx - 2By}{A + B}$

$= \frac{Ax - Ay + Bx - By}{A + B} = \frac{\overline{A + B} \times \overline{x - y}}{A + B} = x - y$. If the bodies move in the oppoſite directions, B's velocity $-$ A's $= x + y$, which is the difference of the velocities, eſtimated in the ſame direction, before impact.

Cor. 5. If p and q be the velocities of A and B after impact, $Ax^2 + By^2 = Ap^2 + Bq^2$. For if the bodies move in the ſame direction, by the third law of motion $Ax + By = Ap + Bq$, alſo $x - y = q - p$, hence

$$\left.\begin{array}{l} A \times \overline{x - p} = B \times \overline{q - y} \\ \quad x + p = \quad q + y \\ \hline \therefore A \times \overline{x^2 - p^2} = B \times \overline{q^2 - y^2} \end{array}\right\}$$

, hence $Ax^2 + By^2 = Ap^2 + Bq^2$. If the bodies move in oppoſite directions, the ſame concluſion follows from the ſame principles.

49. If the bodies be imperfectly elaſtic, and m : n :: the force with which they are compreſſed together till they acquire a common velocity, or perfect elaſticity, : the force with which they ſeparate, or their imperfect elaſticity; then if they move in the ſame direction, A's velocity after impact will be $x - \frac{m + n}{2m} \times \frac{2Bx - 2By}{A + B}$, and B's will be $y + \frac{m + n}{2m} \times \frac{2Ax - 2Ay}{A + B}$. If they move in oppoſite directions, the ſigns of the terms where y enters muſt be changed.

Hence if we have the magnitudes of the bodies A and B, and their velocities x and y before impact, and their velocities after, we ſhall have two equations from which we can determine the ratio of $m : n$, or the degree of elaſticity of the bodies. The velocities after may be found by letting the bodies vibrate through equal arcs and meet at the loweſt point, and then meaſuring the arc each deſcribes after impact. Since the velocity is as the chord of the arc, if we meaſure only the chords in their deſcent and aſcent it will give their relative velocities, which will be ſufficient. If A ſtrike B at reſt and be equal to it, B's velocity after $= \frac{m + n}{2m} \times x$; hence A's velocity before : B's after :: x : $\frac{m + n}{2m} \times x$ :: $2m : m + n$ :: the chord a deſcribed by A : the chord b deſcribed by B; hence $m : n :: a : 2b - a$.

ON

ON THE CENTERS OF PERCUSSION, SPONTANEOUS ROTATION, OSCILLATION AND GYRATION OF BODIES.

Def. 1. The center of *percuſſion* is that point in a body revolving about an axis, at which, if it ſtruck an immoveable obſtacle, all its motion would be deſtroyed, or it would incline neither way.

From this def. it appears, that the point of ſuſpenſion is not affected by the ſtroke.

2. The center of *ſpontaneous rotation* is that point which remains at reſt the inſtant a body is ſtruck, or about which the body begins to revolve.

Hence the center of ſpontaneous rotation is the ſame as the center of ſuſpenſion correſponding to the center of percuſſion, the center of percuſſion being the point where the body is ſtruck. For the action of the body againſt the immoveable obſtacle in the center of percuſſion muſt have the ſame effect upon the body as if the body had been at reſt and the obſtacle had ſtruck the body, in which latter caſe the center of ſuſpenſion would not be affected, and therefore it becomes the center of ſpontaneous rotation.

3. The center of *oſcillation* is that point in a vibrating body at which, if a corpuſcle were ſuſpended, it would vibrate in the ſame time the body does.

4. The center of *gyration* is that point in a body, into which, if the whole quantity of matter were collected, the ſame moving force would generate in it the ſame angular velocity.

50. If a body revolve about an axis, the effect of any particle p to reſiſt, by its inertia, the communication of motion to any point, is as the particle multiplied into the ſquare of its diſtance from the axis.

For the inertia of any particle not acting at any mechanical advantage or diſadvantage to oppoſe the communication of motion, is as its velocity multiplied into its quantity of matter p, or in this caſe as $d \times p$, if d be the diſtance of the particle from the axis, the velocity of each particle varying as d. But this inertia acting upon a lever whoſe length is d in oppoſition to a force acting at any other point, the effect of the inertia, by the property of the lever, will be $d^2 \times p$.

51. If any number of bodies A, B, C, revolve about an axis at the reſpective diſtances a, b, c, and x be the diſtance of the center of gyration

 from

from the axis, then $x = \sqrt{\frac{a^2 \times A + b^2 \times B + c^2 \times C}{A + B + C}}$.

For the inertia of $A + B + C$ placed at the diſtance x from the axis is $x^2 \times \overline{A + B + C}$; now as the moving force is the ſame, the ſame angular velocity will be generated when the inertia is the ſame; hence $x^2 \times \overline{A + B + C} = a^2 \times A + b^2 \times B + c^2 \times C$, therefore $x = \sqrt{\frac{a^2 \times A + b^2 \times B + c^2 \times C}{A + B + C}}$. If the axis paſs through the center of gravity it is called the *principal* center of gyration.

If a ſlender rod whoſe length $= a$ revolve about one end, the diſtance of the center of gyration from that end $= a\sqrt{\frac{1}{3}}$. Or if it revolved about its center, and a were equal to half its length, the diſtance of the center of gyration from the center would be the ſame. If a circle revolve in its own plane about its center, or a cylinder about its axis, and $r =$ the radius, the diſtance of the center of gyration from the center or axis $= r\sqrt{\frac{1}{2}}$. If a globe whoſe radius is r revolve about one of its diameters, the diſtance of the center of gyration from the center $= r\sqrt{\frac{2}{5}}$.

Cor. 1. If a circle be deſcribed about the center of gravity, and any point of its periphery be made the axis of rotation, the diſtance from it to the center of gyration will remain the ſame, the plane of rotation continuing the ſame. This appears from prop. 41.

Cor. 2. To find what quantity of matter Q muſt be placed at any other diſtance d from the axis ſo that the inertia may remain the ſame, we have $d^2 \times Q = x^2 \times \overline{A + B + C}$, hence $Q = \frac{x^2}{d^2} \times \overline{A + B + C}$.

52. If a body vibrate about an axis by the force of gravity, the diſtance of the center of oſcillation from the axis is equal to the ſum of the products of each particle multiplied into the ſquare of its diſtance from the axis, divided by the body multiplied into the diſtance of the center of gravity from the axis. The center of percuſſion is the ſame as the center of oſcillation.

Let

Let LM (fig. 1.) be a plane paſſing through the center of gravity G of the body, perpendicular to the axis of vibration, on which the body is orthographically projected; O the center of oſcillation in the line CG produced; $a, b,$ &c. the conſtituent particles of the body thus projected, Cm parallel to the horizon, and draw Om, Gg, aq, bp, &c. perpendicular to Cm. Now as the angular velocity of each particle of the body is not altered by this projection, we have by $a \times aC^2 + b \times bC^2 + \&c. =$ the inertia of the whole body; alſo if O repreſent a particle at O, $O \times OC^2 =$ the inertia of that particle. Now it appears from prop. 23. that obſ. 5. prop. 22. is true in general, if you aſſume the perpendicular diſtances from the fulcrum inſtead of the real diſtances; therefore $a \times qC + b \times pC + \&c. =$ the effect of gravity to turn the whole body about C, and $O \times mC =$ the effect of gravity to turn the particle O about C. Hence, that the ſame angular velocity may be generated in both caſes, the accelerative forces muſt be in proportion to the reſpective inertias; that is $a \times qC + b \times pC + \&c. : O \times mC ::$ $a \times aC^2 + b \times bC^2 + \&c. : O \times OC^2$, therefore $OC^2 = \frac{\overline{a \times aC^2 + b \times bC^2 + \&c.} \times mC}{a \times qC + b \times pC + \&c.} = \frac{\overline{a \times aC^2 + b \times bC^2 + \&c.} \times mC}{\overline{a + b + \&c.} \times Cg}$; but $CO : mC :: CG : Cg$, $\therefore \frac{CO}{CG} = \frac{mC}{Cg}$; hence $OC = \frac{a \times aC^2 + b \times bC^2 + \&c.}{\overline{a + b + \&c.} \times CG}$. The center of oſcillation thus found being independent of the line Cm, ſhows that it is a fixed point for every poſition of the body. Hence if a body could have all its matter concentrated in O and be ſuſpended at C, it would perform all its vibrations in the ſame time the body does. Hence any body LM thus vibrating, may be conſidered as a pendulum whoſe length is CO, ſo far as regards the time of vibration.

The center of oſcillation of a rod, vibrating about one end, is two thirds of its length from that end. If a ſphere be ſuſpended at the diſtance of d from its center, and r be its radius, the diſtance from the center of ſuſpenſion to the center of oſcillation $= d + \frac{2r^2}{5d}$. Hence if that diſtance a be given, and alſo r, we have $d = \frac{a}{2} \pm \sqrt{\frac{a^2}{4} - \frac{2r^2}{5}}$, which ſhows that there are two points by which it may be ſuſpended to vibrate in the ſame time. When $\frac{a^2}{4} = \frac{2r^2}{5}$, or $a = r\sqrt{\frac{8}{5}}$, there is only one value of d which $= \frac{a}{2} = r\sqrt{\frac{2}{5}}$, which gives the point of ſuſpenſion the center of gyration, in which caſe a is the leaſt poſſible, for if a be aſſumed leſs, d becomes impoſſible. This is agreeable to what is ſhown in a ſubſequent note to this prop. Hence we know $d - r$ the diſtance from the ſurface at which it muſt be ſuſpended to make the length of the pendulum $= a$, which is more convenient than meaſuring

ſuring from the center. If a be not very ſmall in compariſon with r, we may take $d = a$ without any ſenſible error. The diameter of the ball made uſe of in our experiments is 1,6 in. therefore the diſtance of the point of ſuſpenſion from the ſurface $= a - 0,8$ in.

If a rod in the form ⊥ whoſe baſe = 20 in. and perpendicular = 12, the perpendicular biſecting the baſe, be ſuſpended at the upper end and vibrate in its own plane, the diſtance from the point of ſuſpenſion to the center of oſcillation = 13,21 inches; but if it vibrate perpendicular to its plane, the diſtance of the centers = 11,08 inches. If the rod be ſuſpended at the diſtance of 6 inches from the top, the diſtance of the point of ſuſpenſion from the center of oſcillation will be 12,75 inches in the former caſe, and 7,2 inches in the latter. Hence by the laſt obſervation the diſtance of the points of ſuſpenſion from the ſurface of the ſphere which ſhall vibrate in the ſame times are 12,41; 10,28; 11,95 and 6,4 inches reſpectively.

The diſtance of the center of gyration from the axis of motion, is a mean proportional between the diſtances of the centers of oſcillation and gravity from the ſame axis.

Hence the time of vibration will be the leaſt poſſible when the axis paſſes through the principal center of gyration. For if x and $a =$ the diſtances from the center of gravity to the centers of ſuſpenſion and gyration, then $x : x + a :: x + a : \frac{\overline{x+a}^2}{x}$ the length of the pendulum, which is the leaſt when $x = a$.

The product of the diſtances of the center of gravity from the axis of ſuſpenſion and center of oſcillation is a conſtant quantity for the ſame plane of vibration; if therefore the center of oſcillation be made the point of ſuſpenſion, the point of ſuſpenſion becomes the center of oſcillation. Hence alſo it follows, that if upon the plane of vibration paſſing through the center of gravity of any body, two circles be deſcribed with the center of gravity as their center and radii equal to the diſtances of the center of gravity from the point of ſuſpenſion and center of oſcillation, the body ſuſpended from any point in the periphery of either circle, will perform its vibration in the ſame time. Hence there are an indefinite number of points in the ſame body, by which, if the body were ſuſpended, the time of vibration would remain the ſame.

Hence alſo if the point of ſuſpenſion and center of oſcillation be given in one caſe, and alſo any other point of ſuſpenſion, the center of oſcillation will be known, and conſequently the length of the pendulum, the plane of vibration remaining the ſame. For example, if a ſlender rod 36 inches long be ſuſpended at one end, the diſtance of the center of oſcillation will be 24 inches from it, and their diſtances from the center of gravity 18 and 6 inches reſpectively; hence if the ſame rod be ſuſpended 10,39 inches from the center of gravity, then will $\frac{18 \times 6}{10,39} = 10,39$ inches, be the diſtance from the center of gravity to the center of oſcillation; hence the length of the pendulum = 20,78 inches. Here the principle center of gyration is the point

of

of ſuſpenſion, conſequently the time of vibration is the leaſt poſſible. If the rod be ſuſpended 5,2 inches from the center of gravity, the length of the pendulum = 26,17 inches. If it be ſuſpended 3,25 in. from the center of gravity, the length of the pendulum = 36,48 in.

If the ſame body vibrate about an axis at the ſame diſtance from the center of gravity but in a different poſition, the time of vibration will not be the ſame, unleſs the ſum of the products of each particle × the ſquare of its diſtance from the axis remains the ſame.

To find the center of oſcillation of an irregular body ſuſpended at any point, hang up a ſimple pendulum, that is a ſmall globe ſuſpended by a ſtring, and adjuſt its length till it vibrates in the ſame time the body does, and the length of that pendulum is equal to the diſtance of the point of ſuſpenſion from the center of oſcillation of the body. Or two thirds of the length of a ſlender rod ſuſpended at one end and vibrating in the ſame time, will give the ſame.

If a body, inſtead of revolving about a center be made to move parallel to itſelf by being conceived to be ſuſpended at an infinite diſtance, the centers of gravity, oſcillation and percuſſion become the ſame.

53. If a body vibrate, and every particle be attracted to a center by a force which varies as its diſtance from it, the center of oſcillation will be the ſame as when it is acted upon by a conſtant force acting in parallel lines. If the force vary in any other ratio, the center of oſcillation will not be the ſame, nor will it continue a fixed point for a whole vibration, but will vary as the poſition of the body varies.

54. If two bodies p and q hang upon a lever at the diſtances m and n, and p deſcend, the preſſure upon the axis $= \frac{\overline{mp - nq}^2 \times pq}{m^2p + n^2q}$.

Let a and c be the diſtances of the centers of oſcillation and gravity from the center of ſuſpenſion; then as the whole ſyſtem revolves with the ſame angular velocity as if the whole quantity of matter $p + q$ were placed at the diſtance a from the fulcrum, and as the accelerative forces are as the velocities generated in a given time, or as the diſtances from the fulcrum, we have $a : c :: p + q : \frac{c \times \overline{p + q}}{a}$ the force with which the

center

center of gravity descends, and which $= \frac{\overline{mp - nq}^2}{m^2p + n^2q}$ because $a = \frac{m^2p + n^2q}{mp - nq}$ and $c = \frac{mp - nq}{p + q}$; but the force with which the center of gravity descends is that by which the pressure upon the axis is diminished; hence the pressure upon the axis $= p + q - \frac{\overline{mp - nq}^2}{m^2p + n^2q} = \frac{\overline{m + n}^2}{m^2p + n^2q} \times pq.$

Cor. If $m = n$, the pressure $= \frac{4pq}{p+q}$. If $p = 4$ oz. $q = 3$ oz. the pressure $= 6$ oz. 17 drs. 3 grs.

ON THE MOTION OF BODIES ACTED UPON BY UNIFORMLY ACCELERATING FORCES.

Def. A force is said to be uniformly accelerating, when the quantity of acceleration continues to be the same in the same time, or when equal increments of velocity are generated in equal times.

55. If a body move uniformly with the velocity V for the time T, the space described $= TV$.

In mechanics we measure the velocity by the space which a body describes in one second, supposing the velocity to be continued uniform for that time; also the time is estimated in seconds, and the space in feet. Hence, as the spaces must be in proportion to the times when the velocity is uniform, $1'' : T' :: V : TV$ the space described. It may perhaps appear to be improper to multiply time and velocity, two heterogeneous quantities, together; but it must be observed, that T, in the expression TV, is an abstract number, being the quotient of T' divided by $1''$; the abstract number therefore by which V is multiplied is always the same as that which denotes the number of seconds in the given time.

A quantity is said to be given when it continues the same whilst the other quantities with which it is connected vary. Thus, if the time remain the same whilst the velocity varies, or if two bodies move with different uniform velocities for the same time, the time is said to be given, and the spaces will be in proportion to the velocities.

56. The space described in the last proposition may be represented by the area of a right angled parellelogram, one of whose sides represents the time and the other the velocity.

For

For its area is equal to the product of one side × into the other.

57. If the force of gravity be denoted by unity, and F be any other uniformly accelerative force compared with it, also if v be the velocity generated by gravity in one second, and V be the velocity generated by the force F in the time T, then $V = FVv$.

For by the same force the velocity generated must be as the time, equal velocities being generated in equal times, hence $1'' : T :: v : Tv$ the velocity generated by gravity in the time T. Also for the same time the velocity generated must evidently be as the force, hence 1 (grav.) : $F :: Tv : V = FTv$. Here F and T become abstract numbers, being equal to the quotients of force divided by force, and time by time. Hence as v is given, V varies as FT.

58. The accelerative force of a body varies as the moving force directly and the quantity of matter to be moved inversely.

For the moving force is in proportion to the quantity of motion generated by it in a given time, or as the velocity × the quantity of matter; therefore the moving force divided by the quantity of matter varies as the velocity, which varies as the accelerative force, when the time is given, by the last prop.

All bodies descend with equal velocities by the force of gravity, and therefore the moving force must be in proportion to the quantity of matter, for to make twice the quantity of matter descend with the same velocity, twice the force must manifestly be applied. We may therefore here estimate the moving force by the quantity of matter to be moved, and make the accelerative force of gravity unity; and then any other accelerative force may be compared with it, by measuring the force of a body by its quantity of matter when it hangs freely down, or by its quantity of matter, when it moves in any other direction, multiplied into the force of gravity in that direction, because the body must have a less force in any other direction as gravity in that direction is less. If two bodies P and Q, P being the greatest, hang over a pulley, the moving force is $P-Q$, that being the quantity which gives motion to the bodies; but this moving force has both bodies to move, and therefore must move them with less velocity, that is, less accelerate them, or the accelerative force will be less, the greater the bodies are, or the accelerative force will, from this cause only, be inversely as $P+Q$. But it is manifest that a greater moving force must, cæteris paribus, give a proportionably greater acceleration. Hence the

whole accelerative force $= \frac{P-Q}{P+Q}$. In this case the force of gravity $= 1$; for if $Q = o$, P falls freely by gravity, and the accelerative force becomes unity.

59. If a body fall from a ſtate of reſt and be acted upon by any uniformly accelerative force F, and T be the time of its acting, V the velocity acquired at the end of that time, and S the ſpace, then S varies as $T \times V$, or as $F \times T^2$; alſo V^2 varies as $F \times S$.

Here F and T are to be conſidered as abſtract numbers, as explained in prop. 57.

Cor. 1. If F be given, that is, if bodies fall and be acted upon by the ſame uniformly accelerating force, then S varies as T^2, and alſo as V^2. Hence if we take a ſucceſſion of times as 1, 2, 3, 4, 5, 6, &c. the ſpaces will be as 1, 4, 9, 16, 25, 36, &c. conſequently the ſpaces deſcribed in the equal ſucceſſive portions of time will be as the odd numbers 1, 3, 5, 7, 9, &c.

Cor. 2. If the time be given, the ſpace varies as the force, or as the velocity generated.

60. If a body fall from a ſtate of reſt and be acted upon by an uniformly accelerative force, and in deſcending through the ſpace S in the time T it acquire a velocity V, the body with the laſt acquired velocity V continued uniform for the time T would paſs over the ſpace $2S$.

In our latitude, a heavy body is found by experiment to deſcend $16\frac{1}{12}$ feet in the firſt ſecond, hence the velocity acquired in that time is $32\frac{1}{6}$ feet in a ſecond. All bodies would fall with the ſame velocity, were it not for the reſiſtance of the air, for in an exhauſted receiver, a guinea and a feather fall from the top to the bottom in the ſame time. Hence the force of gravity acts equally upon every kind of matter.

61. If a body fall freely by the force of gravity through the ſpace S, and T be the time of deſcent and V the velocity acquired, alſo $m = 16\frac{1}{12}$ feet; then

then 1st. $S = mT^2$; 2dly. $T = \sqrt{\frac{S}{m}}$; 3dly. $S = \frac{V^2}{4m}$; 4thly. $V = \sqrt{4mS}$; 5thly. $V = 2mT$, and 6thly. $T = \frac{V}{2m}$.

62. If a body fall freely by any other force F compared with gravity reprefented by unity, then $S = mFT^2$, $V = \sqrt{4mFS}$ and $T = \frac{V}{2mF}$.

63. If M reprefent the moving force, eftimated as explained in prop. 58. and Q the quantity of matter to be moved, then $F = \frac{M}{Q}$; hence by the laft prop. $S = \frac{M}{Q} \times mT^2$, $V = \sqrt{\frac{4mMS}{Q}}$, and $T = \frac{VQ}{2mM}$.

Cor. 1. If two bodies P and Q hang over a pulley, of which P is the greateft, then $P - Q$ is the moving force, and $P + Q$ the quantity of matter to be moved; hence $S = \frac{P-Q}{P+Q} \times mT^2$. But we have not here taken into confideration the quantity of matter in the pulley to be moved; now as the pulley has a rotatory motion, the different parts of which move with different velocities from that of P and Q, except its circumference, we muft not add the quantity of matter in the pulley to $P + Q$ in order to get the whole inertia, but we muft compute by prop. 51. cor. 2. what quantity of matter q placed in the circumference will retard juft as much as the whole pulley; and then $S = \frac{P-Q}{P+Q+q} \times mT^2$; alfo $V = \sqrt{\frac{4m \times \overline{P-Q} \times S}{P+Q+q}}$, and $T = \frac{V \times \overline{P+Q+q}}{2m \times \overline{P-Q}}$.

If

If the body be irregular, q cannot be computed, but may be thus determined by experiment. With two given weights P, Q, obſerve the ſpace S deſcribed in any time T, and then $q = \frac{\overline{P-Q} \times mT^2}{S} - P - Q$.

To take away all reſiſtance, as far as poſſible, the axis of the wheel over which P and Q hang, ſhould lie upon friction wheels, by which the friction becomes inſenſible, and the experiments will anſwer to the theory without any ſenſible difference.

By a variety of experiments to determine the inertia of the friction wheels made uſe of in our experiments, it appears that $q = 2,75$ oz. troy. The machine for theſe experiments was invented by Mr. ATWOOD, and is moſt admirably adapted to the purpoſes for which it was intended, as the whole theory of uniformly accelerating and retarding forces may be proved by it, as the author himſelf has ſhown in his *Theory of rectilinear and rotatory Motion of Bodies*, and in his excellent *Analyſis of a Courſe of Lectures on the Principles of Natural Philoſophy*, read at *Cambridge*. By altering the value of P, Q, S and T, and by ſuppoſing ſome given whilſt the others vary, the truth of all the principles may be experimentally proved. And by taking off weight from P as it aſcends till it becomes leſs than Q, the motion at that inſtant becomes retarded; and from thence every thing relating to uniformly retarding forces may be proved. If in the deſcent of P, ſo much weight be taken from it as to leave it equal to Q, the bodies go on without any acceleration, and it appears that they then deſcribe twice the ſpace which they had before deſcribed in the ſame time. GALILEO, who firſt gave the theory of uniformly accelerating forces, proved the general law, that the ſpaces vary as the ſquares of the times, by letting bodies deſcend upon inclined planes. But on account of the friction of the planes he could not apply the method to the abſolute quantities of ſpace, time and velocity, nor to the caſe where the moving force and the quantity of matter to be moved are different.

Cor 2. If Q lie on an horizontal plane, the moving force is P only, and hence $S = \frac{P}{P+Q} \times mT^2$, neglecting the inertia of the pulley over which the ſtring runs.

Cor. 3. If P hang freely down, and Q lie upon an inclined plane with the ſtring parallel to it, then if the height of the plane be to its length :: $r : s$, the force of Q's deſcent $= \frac{rQ}{s}$, and if P draw Q up, the moving force $= P - \frac{rQ}{s} = \frac{sP - rQ}{s}$, hence $S = \frac{sP - rQ}{s \times \overline{P+Q}} \times mT^2$, neglecting the inertia of the pulley.

64. If a body deſcend down an inclined plane, the accelerative force : the force of gravity :: the

height

height H : the length L, or, as radius : the cofine of the plane's inclination.

Cor. 1. Hence as the force of gravity is conftant at the fame place, the accelerative force varies as the height of the plane directly and length inverfely, and therefore is uniform for the fame plane.

Cor. 2. If we denote the force of gravity of a body by its weight W, its accelerative force upon the plane $= \frac{H}{L} \times W$.

65. If a body defcend down an inclined plane without friction, the velocity acquired is the fame as that down the height.

Hence the velocities down all planes of the fame height are equal, and vary as the fquare roots of the heights.

66. If a body could defcend without friction down feveral inclined planes connected together, and lofe no motion in going from one to another, the velocity acquired would be the fame as that down the height of the whole fyftem.

67. When a body defcends from one inclined plane to another, the velocity is diminifhed in the ratio of radius : the cofine of the angle between the directions of the planes.

68. If a body defcend without friction down any curve, the velocity is the fame as that which would be acquired down its altitude.

Hence if a body defcend in any curve, it will afcend to the fame altitude in whatever curve it may rife.

Hence when a body is fufpended by a ftring and made to vibrate in any curve, the velocity is that which is acquired in falling down the altitude, there being no friction to retard it.

69. If a body defcend down an inclined plane, the time varies as $\frac{L}{\sqrt{H}}$.

Cor.

Cor. Hence when H is given, or two inclined planes have the ſame height, the time varies as the length. Hence the time down the length : the time down the height :: $L : H$; but by prop. 61. the time down H is equal to $\sqrt{\frac{H}{m}}$, hence the time down $L = \frac{L}{H} \times \sqrt{\frac{H}{m}}$.

70. If the diameter of a circle be perpendicular to the horizon, and chords be drawn from either extremity, the times of deſcent down all the chords are equal, and the velocities and accelerative forces will be as the lengths of the chords.

71. The times down ſimilar ſyſtems of inclined planes, ſimilarly ſituated, vary as the ſquare roots of their lengths, ſuppoſing there be no friction, nor any motion loſt at the angles.

Cor. Hence as a curve may be conſidered as the limit to which a rectilinear figure approaches by diminiſhing the length of its ſides and inclination to each other *ſine limite*, the times down ſimilar curves vary as the ſquare roots of their lengths.

Thus far we have conſidered the motion of bodies acted upon by conſtant accelerative forces; the next propoſition contains the general principles of motion when the forces are variable.

72. If a body deſcend in any line by any conſtant or variable force F compared with gravity repreſented by unity, $x =$ the ſpace deſcribed, v the velocity, t the time, and $m = 16\frac{1}{12}$ feet, then

$$v\dot{v} = \pm 2mF\dot{x}; \text{ alſo } \dot{t} = \pm \frac{\dot{x}}{v}.$$

For $\dot{v}$ varies as $F \times \dot{t}$; but $\dot{t}$ varies as $\pm \frac{\dot{x}}{v}$; hence $\dot{v}$ varies as $\pm \frac{F\dot{x}}{v}$, $\therefore v\dot{v}$ varies as $\pm F\dot{x}$. Now by prop. 61. $v^2 = 4mx$ by gravity, hence $v\dot{v} = \pm 2m\dot{x}$, $\therefore v\dot{v} : \pm 1 \times \dot{x} :: 2m : 1$; but $v\dot{v}$ is in a conſtant ratio to $\pm F\dot{x}$; hence if we conſider 1 as the force of gravity, that ratio is $2m : 1$; hence $v\dot{v} : \pm F\dot{x} :: 2m : 1$, conſequently $v\dot{v} = \pm 2mF\dot{x}$. The ſign + muſt be uſed when v and x increaſe or decreaſe together, and − on the contrary.

Alſo as v denotes the ſpace deſcribed uniformly in 1″, and $\dot{x}$ is deſcribed with the ſame velocity in the time $\dot{t}$; $\therefore$, as the velocity is given, v :

$v : 1'' :: \pm \dot{x} : \dot{t} = \pm \frac{\dot{x}}{v}$, where $+$ muſt be uſed when t and x increaſe or decreaſe together, and $-$ on the contrary.

73. If a body deſcend from reſt in a right line towards a center of force, and x be the diſtance from that center; and if at the diſtance d the force $= c$ compared with gravity repreſented by unity, then, if the force vary as the n^{th}. power of the diſtance, $d^n : x^n :: c : \frac{cx^n}{d^n}$ the force at the diſtance x; hence $v\dot{v} = -\frac{2mcx^n\dot{x}}{d^n}$, $\therefore v^2 = \frac{-4mc}{\overline{n+1} \times d^n} \times x^{n+1}$ $+$ cor. but when $x = a$, the greateſt diſtance, $v = o$, hence $v^2 = \frac{4mc}{\overline{n+1} \times d^n} \times \overline{a^{n+1} - x^{n+1}}$, therefore $v = \sqrt{\frac{4mc}{\overline{n+1} \times d^n}} \times \sqrt{a^{n+1} - x^{n+1}}$. Alſo $\dot{t} = -\frac{\dot{x}}{v} =$

$$\frac{-\dot{x}}{\sqrt{\frac{4mc}{\overline{n+1} \times d^n}} \times \sqrt{a^{n+1} - x^{n+1}}},$$

whoſe fluent, which cannot be expreſſed in general, properly corrected, gives t.

ON THE VIBRATION OF PENDULUMS IN THE ARCS OF CIRCLES AND CYCLOIDS, AND THEIR MOTIONS IN CONICAL SURFACES.

74. If a pendulum vibrate in the arc of a circle, it is accelerated by a force which is to the force of gravity,

gravity, as the ſine of its angular diſtance from the loweſt point to radius.

Cor. Hence in the ſame pendulum, the accelerative force is as the ſine of its diſtance from the loweſt point.

75. If a pendulum vibrate in a circular arc, the longer the vibration is the longer will be the time.

It will appear for prop. 81. and cor. prop. 84. that if the accelerating force varies as the diſtance from the loweſt point, all the vibrations will be performed in the ſame time; but here the accelerating force varies in a leſs ratio, for the ſine varies ſlower than the arc; hence by increaſing the arc the force does not increaſe faſt enough to make all the vibrations equal, and therefore the greater arcs having too ſmall a force for that purpoſe, they will be deſcribed in greater times.

In very ſmall arcs, where the ſine and arc are very nearly equal, all the vibrations, as to ſenſe, will be performed in the ſame time.

76. The times in which different pendulums perform vibrations in ſimilar arcs of circles are as the ſquare roots of their lengths.

For by cor. prop. 71. the times vary as the ſquare roots of the arcs, and the arcs are as the radii.

As the length of the pendulum is denoted by the diſtance from the point of ſuſpenſion to the center of oſcillation, the arc deſcribed by the pendulum is always underſtood to be the arc deſcribed by its center of oſcillation.

77. If a pendulum hang at reſt and a body ſtrike it, then having given the quantity of matter in the ſtriking body and in the pendulum, alſo the point of impact of the pendulum, and the circular arc which it deſcribes, the velocity of the ſtriking body may be found.

Let x be the diſtance from the point of ſuſpenſion to the center of gyration of the pendulum, d the diſtance from the point of ſuſpenſion to the point of impact, m the quantity of matter in the pendulum, M the quantity of matter in the body, $v =$ the velocity communicated to the point ſtruck, $V =$ the velocity of the body, then by cor. 2. prop. 51. if a quantity of matter $= m \times \frac{x^2}{d^2}$ be placed at the point of impact,

the

the ſame angular velocity will be generated; hence by the rule for the collision of nonelaſtic bodies, $\overline{M + m \times \frac{x^2}{d^2}} : M :: V : v$, hence $V = \frac{vMd^2 + vmx^2}{Md^2}$. Now to determine v, obſerve what arc the center of oſcillation of the pendulum deſcribes in its aſcent after impact, and find its verſed-ſine r, then the velocity of the center of oſcillation at the loweſt point $= \sqrt{4mr}$ by prop. 61. the velocity down the verſed-ſine being the ſame as that in the arc by prop. 68. Now let b be the diſtance from the point of ſuſpenſion to the center of oſcillation, then $b : d :: \sqrt{4mr} : v = \frac{d\sqrt{4mr}}{b}$. In this manner Mr. ROBINS determined the initial velocity of balls fired from a gun. To the bottom of the pendulum a ribband was fixed which paſſed between two ſteel edges preſſing againſt each other, ſo that the length of the ribband drawn out gave the chord of the arc deſcribed.

78. The force of gravity is exerted upon every body in proportion to its quantity of matter.

For bodies of different quantities of matter deſcribe the ſame arc in the ſame time, and therefore the greater the quantity of matter the greater muſt be the force exerted upon it in the ſame proportion. Weight alſo being the effect of attraction as the cauſe, muſt be a relative quantity, the ſame body weighing differently on different parts of the earth according as the attraction varies.

LEM. If $v =$ the velocity of a body revolving in a circle whoſe radius $= r$, and the force of gravity be repreſented by $32\frac{1}{6}$ feet, the centrifugal force of the body in the circle $= \frac{v^2}{r}$.

For conceive a body to revolve about the earth at its ſurface, then its centripetal force, or the force of gravity at the earth's ſurface, is equal to its centrifugal force. Hence by Sir I. NEWTON's PRINCIPIA Lib. I. ſect. 2. prop. 4. cor. 1. If $R =$ the radius of the earth, and $V =$ the velocity of a body revolving at its ſurface, the force of gravity : $\frac{V^2}{R}$ :: the centripetal, and conſequently the centrifugal, force of the body revolving in the circle whoſe radius $= r$ with the velocity $v : \frac{v^2}{r}$; but $\frac{V^2}{R} = 32\frac{1}{6}$ feet, hence if the firſt term be made equal to the ſecond, the third will be equal to the fourth.

79. To find the time in which a pendulum describes a conical ſurface CAD.

Let AB (fig. 2.) be the altitude, produce BC to e, and let Ce repreſent the centrifugal force by which the pendulum endeavours to recede from B; draw Cf perpendicular to the horizon, and let it repreſent the force of gravity; compound theſe forces into Cg, which we will ſuppoſe to lie in the direction CA, and then the pendulum will manifeſtly be kept in that poſition. Put $m = 32\frac{1}{6}$ feet, and let it repreſent the force of gravity, $v =$ the velocity of the pendulum in the circle $CmDn$, then by the lem. $\frac{v^2}{BC} =$ the centrifugal force; hence $\frac{v^2}{BC} : m :: Ce : Cf :: CB : AB$, conſequently $v = BC \times \sqrt{\frac{m}{AB}}$ the velocity of the pendulum in a ſecond. Put $p = 6{,}283$ &c. then $p \times BC =$ the circumference of the circle $CmDn$; hence $BC \times \sqrt{\frac{m}{AB}} : p \times BC :: 1'' : p \times \sqrt{\frac{AB}{m}}$ the time of deſcribing the conical ſurface.

Cor. 1. Hence the time of a revolution $p \times \sqrt{\frac{AB}{m}} : \sqrt{\frac{4AB}{m}}$: the time of deſcent through $2AB :: p : 2 ::$ the circumference of a circle : its diameter.

Cor. 2. Hence if the altitude be given, the time of a revolution will be given. If $AB = 9{,}735$ in. the time of a revolution $1''$.

Cor. 3. Hence if the angle CAD be indefinitely ſmall, the time of deſcribing the conical ſurface is equal to twice the time in which the pendulum would vibrate through the diameter CD.

If in this caſe $CA = 3{,}245$ feet, the time of revolution $= 2''$.

DEFINITION.

If a circle roll upon a ſtraight line, any point of its periphery will deſcribe a curve called a *cycloid*.

Lem. 1. If a circle be deſcribed upon the axis of a cycloid, and an ordinate be drawn from the axis parallel to the baſe, the part of the ordinate intercepted between the circle and cycloid will be equal to the chord of the circle drawn from the point where the ordinate cuts the circle to the vertex.

2. The abovementioned chord of the circle is parallel to a tangent to the cycloid at the point where the ordinate meets it.

3. The cycloidal arc intercepted between the vertex and the point where the ordinate meets it, is double to the chord of the circle mentioned in lem. 1.

80. If two equal ſemicycloids be joined at their

baſe

base and have their vertices downwards and axes vertical, and a pendulum equal to the length of one of them be suspended from the point where they touch and vibrate between them, it will describe a cycloid.

Mr. ATWOOD, in his Syllabus of a Course of Lectures read in this University, observes, that this is true only upon supposition that the whole mass of the pendulum is concentrated in a point, for it cannot otherwise take place, because as the string varies in its length the center of oscillation of a body of any magnitude will vary. The property therefore of a pendulum thus vibrating, that it performs all its vibrations in the same time, is not true, and therefore it cannot be so far a true measure of time. Pendulums therefore vibrating in circular arcs are now always used, for the same arcs will always be described in the same time. One principal source of error in a pendulum thus vibrating is, that the different temperatures of the air will alter the length of the pendulum. To prevent this, Mr. HARRISON invented a pendulum composed of rods of iron and brass so framed together, that the brass expands upwards whilst the iron expands downwards, and by thus counteracting each other, the length of the pendulum is preserved very nearly the same in all temperatures, Different methods have also been invented to answer the same purpose.

81. If a pendulum describe any arc of a cycloid, its velocity at any point varies as the right sine of a circular arc, whose diameter is the arc of the cycloid described, and versed-sine the space passed over.

82. The accelerating force of a pendulum vibrating in a cycloid varies as the arc of its distance from the lowest point.

83. The time in which a pendulum vibrates in a cycloid : the time a body would descend down the axis :: the circumference of a circle : its diameter.

84. If L be the length of a pendulum, and F the force of gravity, the time T of vibration varies as $\sqrt{\frac{L}{F}}$.

Cor.

Cor. Hence all the vibrations in the ſame cycloid are performed in the ſame time.

In our latitude a pendulum 39,2 inches long is found to vibrate in one ſecond; hence if the force be given, as it will be for the ſame latitude, we have $\sqrt{39,2} : \sqrt{L} :: 1'' : T'' = \sqrt{\frac{L}{39,2}}$ the time of vibration of a pendulum whoſe length is L, L being taken in inches. If gravity be repreſented by unity, and F repreſent any other force acting on the pendulum, then $\sqrt{\frac{39,2}{1}} : \sqrt{\frac{L}{F}} :: 1 : T' = \sqrt{\frac{L}{39,2 \times F}}$ the time of vibration. This may be applied to compare the vibrations of pendulums on different planets, F being taken to repreſent the reſpective forces of gravity on their ſurfaces. At the diſtance of n radii from the earth's ſurface $F = \frac{1}{n^2}$, and $T = n\sqrt{\frac{L}{39,2}}$; and at the diſtance of $\frac{1}{n}$ of the radius within, $F = \frac{1}{n}$, and $T = \sqrt{\frac{n \times L}{39,2}}$. Hence if n be infinite, or $\frac{1}{n} = o$, T is infinite; for in this caſe, as the point of ſuſpenſion is at the center of the earth, the direction of gravity is in the direction of the pendulum, and therefore if the pendulum be made to vibrate it muſt continue to revolve in a circle.

85. If L be the length of a pendulum, and $D : C ::$ the circumference of a circle : its diameter, then the ſpace S through which a body falls in the time of vibration : the time down $\frac{1}{2}L :: C^2 : D^2$.

Hence if S be given L will be known, and if L be given S will be known.

ON THE MOTION OF PROJECTILES.

86. If the force of gravity were conſtant and acted in parallel lines, and there were no reſiſtance from the air, a body thrown in any direction would deſcribe a parabola.

From

From the ſmall diſtances to which we can project bodies upon the earth's ſurface, the want of an uniform gravity, and its not acting in parallel lines, will not ſenſibly cauſe the motion of the body to deviate from a parabola; but the reſiſtance of the air is ſo great, particularly in ſwift motions, the reſiſtance varying as the ſquare of the velocity, and ſometimes in a greater ratio, that no practical concluſions can be drawn from this theory.

87. The velocity V in any point of the parabola is that which would be acquired by a body falling down $\frac{1}{4}$ of the latus rectum L belonging to that point.

Cor. Hence if $m = 16\frac{1}{12}$ feet, $V = \sqrt{mL}$, and the time of a body's deſcent down $L = \frac{V}{m}$. Hence alſo V varies as $\sqrt{L}$. As gravity acts in parallel lines perpendicular to the horizon, the horizontal velocity of a body would not be altered if there were no reſiſtance of the air, and hence a body in any part of its motion would ſtrike an object perpendicular to the horizon with the ſame force.

88. If a body be thrown in any direction, its amplitude upon an horizontal plane varies as the ſine of double the angle of elevetion, its altitude varies as the verſed-ſine of double the angle of elevation, and the time of flight varies as the ſine of the angle of elevation.

89. If v be the velocity of projection, $m = 16\frac{1}{12}$ feet, $s =$ the ſine, $r =$ the verſed-ſine of double the angle of elevation, $t =$ the ſine of the angle of elevation to radius unity, $a =$ the amplitude on an horizontal plane, $h =$ the altitude, $x =$ the time of flight; then will $a = \frac{sv^2}{2m}$, $h = \frac{rv^2}{8m}$ and $x = \frac{tv}{m}$.

Hence the amplitude is the greateſt at the angle of 45° and $= \frac{v^2}{2m}$, or $\frac{1}{2}L$;

$\frac{1}{2}L$; and the amplitudes are equal at angles equally above and below 45°. This would be the case if there were no resistance from the air, but on account of that, the greatest ranges are at an angle less than 45°.

90. If a body be projected upon an inclined plane, the cosine of whose inclination is w, r and s = the sines of the angles which the direction makes with the plane and zenith, v = the velocity of projection, a = the amplitude, h = the altitude, t = the time of flight; then $a = \frac{rsv^2}{mw^2}$, $h = \frac{r^2v^2}{4mw^2}$, and $t = \frac{rv}{mw}$.

We have before observed, that in very quick motions the theory and practice will differ very much. Mr. ROBINS observes, that a 24 pound shot, impelled with its usual charge of powder, meets with an opposition from the air equivalent to 400 lb. which retards the motion so much, that the range at an elevation of 45° would not be above $\frac{1}{5}$ of that given by theory. And by experiments made with a wooden bullet fired at 45°, he found the range to be only 200 yards, whereas, without any resistance, it would have gone 15000 yards. Mr. ROBINS also found that very little advantage was gained by projecting a body with a greater velocity than 1200 feet in a second. He found that a 24 lb. shot, when discharged with a velocity of 2000 feet in a second, will be reduced to 1200 feet in a second in a flight of a little more than 500 yards. In consequence of this quick destruction of velocity, he found that a less projectile velocity at the same angle might carry a ball further than a greater; for the body projected with the greater velocity, when its velocity becomes equal to that of the other projection, has a less angle of elevation, on which account it may go to a less distance from thence, so as to make the whole distance less. No gun to carry far should be charged with powder whose weight is more than $\frac{1}{6}$ or $\frac{1}{5}$ of the weight of the ball, for that will give field pieces a velocity of 1200 feet in a second. In a battering piece, to fire at a near object, the charge should be about $\frac{1}{3}$ of the weight of the ball. When the velocity of the body is greater than about 1100 or 1200 feet in a second, the resistance appears to be nearly 3 times greater than it ought, if it varied only as the square of the velocity. On this Mr. ROBINS makes the following curious remark. "The velocity at which the variation of the law of resistance takes place, is nearly the same as that with which sound moves. Indeed if the treble resistance in the greater velocities is owing to a vacuum being left behind the resisted body, it is not unreasonable

reaſonable to ſuppoſe, that the celerity of ſound is the very leaſt degree of celerity with which a projectile can form this vacuum, and can in ſome ſort avoid the preſſure of the atmoſphere on its hinder parts. It may perhaps confirm this conjecture to obſerve, that if a bullet, moving with the velocity of ſound, does really leave a vacuum behind it, the preſſure of the atmoſphere on its fore part is a force about 3 times as great as its reſiſtance, computed by the laws obſerved for ſlow motions." Now we may here obſerve, that, taking with Mr. Cotes the height of an homogeneous atmoſphere 29254 feet, if we ſuppoſe the velocity with which the air would ruſh into a vacuum, is, like other fluids, that which is acquired by falling through the whole height 29254 feet of an uniform atmoſphere, that velocity will be found to be 1368 feet in a ſecond, which differs but a little from that ſuppoſed by Mr. Robins. That bodies are therefore reſiſted in air as the ſquares of their velocities is not true, if the velocity be greater than about 1200 feet in a ſecond; after that the reſiſtance is almoſt 3 times greater than this law gives it.

91. If a body be projected, and has a rotatory motion about an axis perpendicular to the horizon, it will deviate from a plane perpendicular to the horizon.

Mr. Robins was the firſt who confirmed this by experiment. Balls from guns are frequently found to deviate to the right or left of the direction of projection. Rifle barrels are therefore uſed to prevent this effect. For the deflecting power is found to act on that ſide where the rotatory and progreſſive motions conſpire. Now in rifle barrels, the axis of the ball's motion lies in the plane of projection, therefore the rotatory motion being perpendicular to the progreſſive motion, no effect is produced. But in every other poſition of the axis there muſt be a deflection, and ſo much the greater by how much the more the axis deviates from the plane of projection towards the horizon.

ON THE MOTION OF BODIES AFFECTED BY FRICTION.

92. If one hard body move upon another, its friction will be an uniformly retarding force.

For if a body be drawn upon an horizontal plane by another hanging perpendicularly, the ſpaces appear to vary as the ſquares of the times, therefore the accelerative force muſt be uniform. Now without friction the acceleration would be uniform, and as it appears to be uniform with it, the difference, or retardation from the friction muſt be uniform.

If

If the body be covered with cloth, woollen, &c. it appears that the resistance increases with the velocity, as the space increases in a much less ratio than the square of the time.

93. If M = the moving force of the body hanging perpendicularly, expressed by its weight, F = the friction considered as equivalent to a weight without inertia drawing the body back upon the horizontal plane, W = the weight of the body upon the plane, S = the space described by M in t seconds, and $m = 16\frac{1}{12}$ feet; then by prop. 63. $\frac{M-F}{M+W} \times mt^2 = S$, hence $F = M - \frac{M+W}{mt^2} \times S$.

94. The quantity of friction increases in a less ratio than the weight of the body.

If we increase M and W in the same ratio, then if F increased in the same ratio, the same space would be described in the same time; but by thus increasing M and W, it appears by experiment that S is increased in the same time, therefore F must have increased in a less ratio than M and W.

95. The friction of a body does not continue the same when it has different surfaces applied to the plane on which it moves, but the smallest surface will have the least friction.

For in the same time with the same moving force, the body describes the greatest space when it moves upon its least surface. The experiments by which these propositions were originally proved may be seen in the *Phil. Trans.* 1784.

The experiments which have been made by all authors whom I have seen, have been thus instituted. To find what moving force would *just* put a body in motion; and they have concluded from thence, that the accelerative force was then equal to the friction; but it is manifest, that any force which will put a body in motion must be *greater* than the force which opposes the motion; and hence, if there were no other objection than this, the quantity of friction could not be very accurately determined. But there is another circumstance which totally destroys the experiment, so far as it tends to show the quantity of friction, which is, the strong cohesion of the body to the plane when it lies at rest; and this

this is confirmed by the following experiments. Firſt, A body of $12\frac{3}{4}$ oz. was laid upon an horizontal plane, and then loaded with a weight of 8 lb. and ſuch a moving force was applied as would, when the body was juſt *put* in motion, continue that motion without any acceleration, in which caſe the friction muſt be juſt equal to the accelerative force. The body was then ſtopped, when it appeared, that the ſame moving force which had *kept* the body in motion before, would not *put* it in motion, and it was found neceſſary to take off $4\frac{1}{2}$ oz. from the body before the ſame moving force *would* put it in motion; it appears, therefore, that this body, when laid upon the plane at reſt, acquired a very ſtrong coheſion to it. Secondly, A body whoſe weight was 16 oz. was laid at reſt upon the horizontal plane, and it was found that a moving force of 6 oz. would juſt *put* it in motion; but that a moving force of 4 oz. *would*, when it was juſt put in motion, *continue* that motion without any acceleration, and therefore the accelerative force muſt *then* have been equal to the friction, and not when the moving force of 6 oz. was applied.

From theſe experiments therefore it appears, that the coheſion was ſo very conſiderable in proportion to the friction when the body was in motion, that it muſt by no means be neglected in our enquiries reſpecting the quantity of friction. All the concluſions therefore deduced from the experiments, which have been inſtituted to determine the friction from the force neceſſary to *put* a body in motion (and I have never ſeen any deſcribed but upon ſuch a principle) have manifeſtly been totally falſe; as ſuch experiments only ſhew the reſiſtance which ariſes from the coheſion and friction conjointly.

96. Let *e*, *f*, *g*, (fig. 3.) repreſent either a cylinder, or that circular ſection of a body on which it rolls down the inclined plane *CA* in conſequence of its friction, to find the time of deſcent and the number of revolutions.

As it has been proved in prop. 94. that the friction of a body does not increaſe in proportion to its weight or preſſure, we cannot therefore, by knowing the friction on any other plane, determine the friction on *CA*; the friction therefore on *CA* can only be determined by experiments made upon *that* plane, that is, by letting the body deſcend from reſt, and obſerving the ſpace deſcribed in the firſt ſecond of time; call that ſpace *a*, and then, as by prop. 92. friction is an uniformly retarding force, the body muſt be uniformly accelerated, and conſequently the whole time of deſcent in ſeconds will be $= \sqrt{\frac{AC}{a}}$. Now to determine the number of revolutions, let *s* be the center of oſcillation to the point of ſuſpenſion *a*; then, becauſe no force acting at *a* can affect the motion of the point *s*, that point, notwithſtanding the action of the friction at *a*, will always have a motion parallel to *CA* uniformly ac-

 celerated

celerated by a force equal to that with which the body would be accelerated if it had no friction; hence, if $2m = 32\frac{1}{6}$ feet, the velocity acquired by the point s in the first second will be $= \frac{2m \times CB}{CA}$; now the excess of the velocity of the point s above that of r (r being the center) is manifestly the velocity with which s is carried about r; hence the velocity of s about the center $= \frac{2m \times CB}{CA} - 2a = \frac{2m \times CB - 2a \times CA}{CA}$, consequently, $rs : ra :: \frac{2m \times CB - 2a \times CA}{CA} : \frac{2m \times ra \times CB - 2a \times ra \times CA}{rs \times CA}$ = the velocity with which a point of the circumference is carried about the center, and which therefore expresses the force which accelerates the rotation; now as $2a$ expresses the accelerative force of the body down the plane, and the spaces described in the same time are in proportion to those forces, we have $2a : CA :: \frac{2m \times ra \times CB - 2a \times ra \times CA}{rs \times CA} : \frac{m \times ra \times CB - a \times ra \times CA}{a \times rs}$ the space which any point of the circumference describes about the center in the whole time of the body's descent down CA; which being divided by the circumference $p \times ra$ (where $p = 6{,}283$ &c.) will give $\frac{m \times BC - a \times AC}{p \times a \times rs}$ for the whole number of revolutions required.

Cor. 1. If $a \times CA = m \times BC$, the number of revolutions $= 0$, and therefore the body will then only slide; consequently the friction vanishes.

Cor. 2. Let $a'r's'$ (fig. 4.) be the next position of $a\,r\,s$, and draw $t\,r'b$ parallel to sa, then will $s\,t$ represent the retardation of the center r arising from friction, and $a\,b$ will represent the acceleration of a point of the circumference about its center; hence the retardation of the center : acceleration of the circumference about the center :: $s't : ab$ :: (by sim. Δ's) $tr' : br$ :: $rs : ra$.

Cor. 3. If a coincide with a, the body does not *slide* but only *roll*; now in this case $ss' : rr' :: aa' : ar$; but as ss' and rr' represent the ratio of the velocities of the points s and r, they will be to each other as $\frac{2m \times BC}{CA} : 2a$ or as $m \times CB : a \times CA$; hence, when the body *rolls* without *sliding*, $as : ar :: m \times CB : a \times CA$.

Cor. 4. The time of descent down CA is $= \sqrt{\frac{AC}{a}}$; but by the last cor. when the body *rolls* without *sliding*, $a = \frac{m \times ra \times BC}{sa \times AC}$, hence the time of descent in that case $= AC \sqrt{\frac{sa}{m \times ra \times BC}}$; now the time

of

of defcent, if there were no friction, would be $= \frac{AC}{\sqrt{m \times BC}}$; hence the time of defcent, when the body *rolls* without *fliding*: time of free defcent :: $\sqrt{sa} : \sqrt{ra}$. If the body be a cylinder, the time of rolling without fliding $= 0,305\, AC \times \sqrt{\frac{1}{BC}}$. If $BC = 1$ ft. $AC = 3$ ft. 3,36 in. the time of defcent $= 1''$. If $AC = 4$ ft. $BC = 4,46$ in. the time $= 2''$. The dimenfions of the cylinder are of no confequence.

Cor. 5. By the laft cor. it appears, that when the body juft *rolls* without *fliding*, or when the friction is juft equal to the accelerative force, the time of defcent $= AC \sqrt{\frac{sa}{m \times ra \times BC}}$; now it is manifeft, that the time of defcent will continue the fame, if the friction be increafed, for the body will ftill freely roll, as no increafe of the friction acting at a can affect the motion of the point s.

If the body be projected from C with a velocity, and at the fame time have a rotatory motion, the time of defcent and the number of revolutions may be determined from the common principles of uniformly accelerated motions, as we have already inveftigated the accelerative force of the body down the plane and of its rotation about its axis; it feems therefore unneceffary to add any thing further upon that fubject.

97. Let a body be projected on an horizontal plane LM (fig. 5.) with a given velocity, to determine the fpace through which the body will move before it ftops, or before its motion becomes uniform.

Case I. 1. Suppofe the body to have no rotatory motion when it begins to move; and let $a =$ the velocity of projection *per* fecond meafured in feet, and let the retarding force of the friction of the body, meafured by the velocity of the body which it can deftroy in one fecond of time, be determined by experiment and called F, and let x be the fpace through which the body would move by the time its motion was all deftroyed, when projected with the velocity a, and retarded by a force F; then, from the principles of uniformly retarded motion, $x = \frac{a^2}{2F}$; and if $t =$ time of defcribing that fpace, we have $t = \frac{a}{F}$, and hence the fpace defcribed in the firft fecond of time $= \frac{2a - F}{2}$. Now it is manifeft, that when the rotatory motion of the body about its axis is equal to its progreffive motion, the point a will be carried backwards by the

former motion, as much as it is carried forwards by the *latter*; consequently the point of contact of the body with the plane will then have no motion in the direction of the plane, and hence the friction will at that instant cease, and the body will continue to *roll* on uniformly without *sliding* with the velocity which it has at that point. Put therefore $z =$ the space described from the commencement of the motion till it becomes uniform, then the body being uniformly retarded, the spaces from the end of the motion vary as the squares of the velocities; hence $\frac{a^2}{2F} : a^2$ $(:: 1 : 2F)$ $:: \frac{a^2}{2F} - z : a^2 - 2Fz =$ square of the progressive velocity when the motion becomes uniform; therefore the velocity destroyed by friction $= a - \sqrt{a^2 - 2Fz}$; hence, as the velocity generated or destroyed in the same time is by proposition 57. in proportion to the force, we have by cor. 2. prop. 96. $rs : ra :: a - \sqrt{a^2 - 2Fz} : \frac{ra}{rs} \times \overline{a - \sqrt{a^2 - 2Fz}}$ the velocity of the circumference efg generated about the center, consequently $\sqrt{a^2 - 2Fz} = \frac{ra}{rs} \times \overline{a - \sqrt{a^2 - 2Fz}}$, and hence $z = \frac{\overline{rs^2 + 2rs \times ra} \times a^2}{as^2 \times 2F}$ the space which the body describes before the motion becomes uniform.

2. If we substitute this value of z into the expression for the velocity, we shall have $a \times \frac{ra}{rs}$ for the velocity of the body when its motion becomes uniform; hence therefore it appears, that the velocity of the body, when the friction ceases, will be the same whatever be the quantity of the friction. If the body be the circumference of a circle, it will always lose half the velocity before its motion becomes uniform.

Case II. 1. Let the body, besides having a progressive velocity in the direction LM, have also a rotatory motion about its center in the direction gfe, and let v represent the initial velocity of any point of the circumference about the center, and suppose it first to be less than a; then friction being an uniformly retarding force, no alteration of the velocity of the point of contact of the body upon the plane can affect the quantity of friction; hence the progressive velocity of the body will be the same as before, and consequently the rotatory velocity generated by friction will also be the same, to which if we add the velocity about the center at the beginning of the motion, we shall have the whole rotatory motion; hence therefore, $v + \frac{ra}{rs} \times \overline{a - \sqrt{a^2 - 2Fz}} = \sqrt{a^2 - 2Fz}$, consequently $z = \frac{a^2 \times as^2 - \overline{v \times rs + a \times ra}^2}{2F \times as^2}$ the space described before the motion becomes uniform.

2. If this value of z be substituted into the expression for the velocity,

city, we shall have $\frac{v \times rs + a \times ra}{as}$ for the velocity when the friction ceases.

3. If $v = a$, then $z = o$, and hence the body will continue to move uniformly with the first velocity.

4. If v be greater than a, then the rotatory motion of the point a on the plane being greater than its progressive motion and in a contrary direction, the absolute motion of the point a upon the plane will be in the direction ML, and consequently friction will now act in the direction LM in which the body moves, and therefore will accelerate the *progressive* and retard the *rotatory* motion; hence it appears, that the progressive motion of a body may be accelerated by friction. Now to determine the space described before the motion becomes uniform, we may observe, that as the progressive motion of the body is now accelerated, the velocity after it has described any space z will be $= \sqrt{a^2 + 2Fz}$, hence the velocity acquired $= \sqrt{a^2 + 2Fz} - a$, and consequently the rotatory velocity destroyed $= \frac{ra}{rs} \times \sqrt{a^2 + 2Fz} - a$, hence $v - \frac{ra}{rs} \times \sqrt{a^2 + 2Fz} - a = \sqrt{a^2 + 2Fz}$, therefore $z = \frac{\overline{rs \times v + ra \times a}^2 - a^2 \times as^2}{2F \times as^2}$ the space required.

5. If $a = o$, or the body be placed upon the plane without any progressive velocity, then $z = \frac{rs^2 \times v^2}{2F \times as^2}$.

CASE III. 1. Let the given rotatory motion be in the direction gef; then as the friction must in this case always act in the direction ML, it must continually tend to destroy both the progressive and rotatory motion. Now as the velocity destroyed in the same time is in proportion to the retarding force, and the force which retards the *rotatory* is to the force which retards the *progressive* velocity by cor. 2. prop. 96. as $ra : rs$, therefore if v be to a as ra is to rs, then the retarding forces being in proportion to the velocities, both motions will be destroyed together, and consequently the body, after describing a certain space, will rest; which space, being that described by the body uniformly retarded by the force F, will, from what was proved in case I. be equal to $\frac{a^2}{2F}$.

2. If v bear a greater proportion to a than ra does to rs, it is manifest, that the rotatory motion will not be all destroyed when the progressive is; consequently the body, after it has described the space $\frac{a^2}{2F}$, will return back in the direction ML; for the progressive motion being then destroyed, and the rotatory motion still continuing in the direction gef, will cause the body to return with an accelerated velocity until

until the friction ceaſes by the body's beginning to roll, after which it will move on uniformly. Now to determine the ſpace deſcribed before this happens, we have $rs : ra :: a : \frac{ra \times a}{rs}$ the rotatory velocity deſtroyed when the progreſſive is all loſt; hence $v - \frac{ra \times a}{rs} = \frac{v \times rs - a \times ra}{rs}$ = the rotatory velocity at that time, which being ſubſtituted for v in the laſt article of caſe II. gives $\frac{\overline{v \times rs - a \times ra}^2}{2F \times as^2}$ for the ſpace deſcribed before the motion becomes uniform.

3. If v have a leſs proportion to a than ra has to rs, it is manifeſt, that the *rotatory* motion will be deſtroyed before the *progreſſive*; in which caſe a rotatory motion will be generated in a contrary direction until the two motions become equal, when the friction will inſtantly ceaſe, and the body will then move on uniformly. Now $ra : rs :: v : \frac{v \times rs}{ra}$ the progreſſive velocity deſtroyed when the rotatory velocity ceaſes, hence $a - \frac{v \times rs}{ra} = \frac{a \times ra - v \times rs}{ra}$ = progreſſive velocity when it begins its rotatory motion in a contrary direction; ſubſtitute therefore this quantity for a in the expreſſion for z in caſe I. and we have $\frac{\overline{rs^2 + 2rs \times ra} \times \overline{a \times ra - v \times rs}^2}{as^2 \times ar^2 \times 2F}$ for the ſpace deſcribed after the rotatory motion ceaſes before the motion of the body becomes uniform. Now to determine the ſpace deſcribed before the rotatory motion was all deſtroyed, we have (as the ſpace from the end of a uniformly retarded motion varies as the ſquare of the velocity) $a^2 : \frac{a^2}{2F} :: \frac{\overline{a \times ra - v \times rs}^2}{ra^2} : \frac{\overline{a \times ra - v \times rs}^2}{2F \times ra^2}$, the ſpace that would have been deſcribed from the time that the rotatory velocity was deſtroyed, until the progreſſive motion would have been deſtroyed, had the friction continued to act; hence $\frac{a^2}{2F} - \frac{\overline{a \times ra - v \times rs}^2}{2F \times ra^2} = \frac{2av \times ra \times rs - v^2 \times rs^2}{2F \times ra^2}$ = the ſpace deſcribed when the rotatory motion was all deſtroyed; hence $\frac{\overline{rs^2 + 2rs \times ra} \times \overline{a \times ar - v \times sr}^2}{as^2 \times ar^2 \times 2F} + \frac{2av \times ra \times rs - v^2 \times rs^2}{2F \times ra^2}$ = whole ſpace deſcribed by the body before its motion becomes uniform.

DEFINITION.

The *center of friction* is that point in the baſe of a body on which it revolves, into which if the whole ſurface of the baſe, and the maſs of the body were collected, and made to revolve about the center of the baſe of the given body, the angular velocity deſtroyed by its friction would be equal to the angular velocity deſtroyed in the given body by its friction in the ſame time.

98. To find the center of friction.

Let FGH (fig. 6.) be the baſe of a body revolving about its center C, and ſuppoſe a, b, c, &c. to be indefinitely ſmall parts of the baſe, and let A, B, C, &c. be the correſponding parts of the ſolid, or the priſmatic parts having a, b, c, &c. for their baſes; and P the center of friction. Now it is manifeſt, that the decrement of the angular velocity muſt vary as the whole diminution of the momentum of rotation, cauſed by the friction *directly*, and as the whole momentum of rotation or effect of the inertia of all the particles of the ſolid, *inverſely*; the *former* being employed in diminiſhing the angular velocity, and the *latter* in oppoſing that diminution by the endeavour of the particles to preſerve in their motion. Hence, if the effect of the friction vary as the effect of the inertia, the decrements of the angular velocity in a given time will be equal. Now as the quantity of friction (as has been proved from experiments) does not depend on the velocity, the effect of the friction of the elementary parts of the baſe a, b, c, &c. will be as $a \times aC$, $b \times bC$, $c \times cC$, &c. alſo the effect of the inertia of the correſponding parts of the body will be as $A \times aC^2$, $B \times bC^2$, $C \times cC^2$, &c. Now when the whole ſurface of the baſe and maſs of the body are concentrated in P, the effect of the friction will be as $\overline{a+b+c+\&c.} \times CP$, and of the inertia as $\overline{A+B+C+\&c.} \times CP^2$; conſequently $a \times aC + b \times bC + c \times cC + \&c. : \overline{a+b+c+\&c.} \times CP :: A \times aC^2 + B \times bC^2 + C \times cC^2 + \&c. : \overline{A+B+C+\&c.} \times CP^2$; and hence $CP = \dfrac{\overline{A \times aC^2 + B \times bC^2 + C \times cC^2 + \&c.} \times \overline{a+b+c+\&c.}}{\overline{a \times aC + b \times bC + c \times cC + \&c.} \times \overline{A+B+C+\&c.}} =$ (if $S =$ the ſum of the products of each particle into the ſquare of its diſtance from the axis of motion, $T =$ the ſum of the products of each part of the baſe into its diſtance from the center, $s =$ the area of the baſe, $t =$ the ſolid content of the body) $\dfrac{S \times s}{T \times t}$.

99. Given the velocity with which a body begins to revolve about the center of its baſe, to determine the number of revolutions which the body will make before all its motion is deſtroyed.

Let

Let the friction, expressed by the velocity which it is able to destroy in one second in the body if it were projected in a right line horizontally be determined by experiment, and called F; and suppose the initial velocity of the center of friction P about C to be a. Then conceiving the whole surface of the base and mass of the body to be collected into the point P, and (as has been proved in prop. 97.) $\frac{a^2}{2F}$ will be the space which the body so concentrated will describe before all its motion is destroyed; hence if we put $z = PC$, $p =$ the circumference of a circle whose radius is unity, then will $pz =$ circumference described by the point P; consequently $\frac{a^2}{2pzF} =$ the number of revolutions required.

Cor. If the solid be a cylinder and r be the radius of its base, then $z = \frac{3r}{4}$, and therefore the number of revolutions $= \frac{2a^2}{3prF}$.

100. When a wheel turns upon an axle, the force to overcome the friction is diminished in the ratio of the radius of the wheel to the radius of the axle.

For the force which turns the wheel acts at the circumference of the wheel, and the friction acts at the circumference of the axle; the force therefore acting at a greater distance from the axis acts at so much a greater advantage to overcome the friction.

Hence in friction wheels, where the axle of the wheel to which the weight is applied lies upon the circumference of two other wheels turning upon their axles, the friction is diminished in the ratio of the product of the radii of the wheels to the product of the radii of the axles.

ON WHEEL CARRIAGES.

On plain hard ground.

101. The utility of wheels arises from their turning about their axles.

For it requires a less force to draw the carriage when they are free to turn about their axles, than when they are chained together and cannot turn.

102. If the wheels be all equal and narrow, it re-

requires the ſame weight to draw the carriage, whether it be loaded before or behind.

103. If broad wheels be put on of the ſame ſize and weight, it requires the ſame weight to draw the carriage as for the narrow wheels, at whatever part it is loaded.

104. If two wheels be low and two high, it requires a greater weight to draw the carriage than when all are high.

In this caſe it makes no ſenſible difference which go before. The common opinion therefore that the high wheels drive on the lower when they go forward is not true.

105. If the wheels be all equal, it requires a greater weight to draw the carriage, the leſs the wheels are.

The diſadvantage of ſmall wheels ariſes from hence, that the reſiſtance of the ground, which turns the wheels about, more eaſily overcomes the friction at the axle in a large than a ſmall wheel, becauſe it acts at a greater diſtance. For the mechanical advantage of wheels is, that the reſiſtance which muſt be overcome by a force more than equivalent to it if the wheels could not turn, is overcome by a leſs force in the proportion of the radius of the wheel to the radius of the axle, when the wheels do turn. Hence the diſadvantage of laying the load upon the low wheels, as it increaſes the friction where there is the leaſt power to overcome it. Dr. DESAGULIERS has given a wrong reaſon for the diſadvantage of ſmall wheels; for he ſays, that the ſmaller wheel moving quicker upon the axis than the large one, muſt have ſo much the more friction; whereas it appears by prop. 92. that the friction is the ſame whatever be the velocity. Where the load is but ſmall, and conſequently the friction but ſmall, there is but a ſmall difference between the ſmall and large wheels; but when the load is great the difference becomes conſiderable. The high wheels uſed in theſe experiments were 6 inches diameter, and the low wheels 3, and the carriage weighed about 22 oz. excluſive of the wheels.

On hard ground with obſtacles.

106. If W be the weight of the carriage, and the center of gravity be in the middle; alſo if $r =$

the radius of the wheel and $x=$ the height of the obſtacle, then the power P acting parallel to the horizon which is juſt ſufficient to balance the carriage at the obſtacle without drawing it over $=$

$$\frac{W\times\sqrt{2rx-x^2}}{2r-2x}.$$

For the power may be conceived to be drawing a weight up an inclined plane which is a tangent to the circle at the point where it touches the obſtacle; and as, when that end riſes, the other reſts upon the horizontal plane, the power has to elevate a weight only equal to $\frac{1}{2}W$.

Experiments of this kind are ſubject to inaccuracies which cannot be accounted for. The power will ſometimes hang for ſome time without moving the carriage, and then it will ſuddenly draw the carriage over the obſtacle. Sometimes there will be a difference of $\frac{1}{2}$ oz. out of about 10 oz. in drawing the ſame carriage over the ſame obſtacle, although every care is taken to have all the circumſtances accurately the ſame. Many of the experiments however anſwer very nearly to the theory, nor do any of them differ from it very materially.

The uſe of high wheels in going over obſtacles is very manifeſt from this propoſition, and as carriages are continually going over obſtacles, high wheels will always have the advantage. Moreover in ſinking into holes they have a double advantage, firſt, they do not ſink ſo deep as low ones would, and ſecondly, after ſinking, they aſcend again with leſs power. As, when the center of gravity is in the middle of the carriage, the power has but half its weight to elevate in going over an obſtace, therefore when the load is not in the middle, it throws the center of gravity towards one end, and therefore when that end goes over an obſtacle the power has more than half the weight to raiſe, the preſſure upon each wheel being inverſely as the diſtance of the center of gravity from them. Hence every carriage ſhould be loaded moſt towards the higheſt wheels, by which means leſs than half the weight will be thrown upon the lower wheels, and thus each pair of wheels may be made to require the ſame power to draw them over an obſtacle. The ſame power however that may be neceſſary for one obſtacle will not be ſufficient for another.

If the height of the obſtacle be inconſiderable in reſpect to the radius of the wheel, which is the caſe with the common obſtacles, as ſtones, &c. which carriages uſually meet with, then $P=W\times\sqrt{\frac{x}{2r}}$. Now as each pair of wheels has the ſame obſtacles to go over, x is given, and that P may be given, or that it may require the ſame power for each pair, W muſt vary as $\sqrt{r}$; now the weight ſupported by each wheel is inverſely as its diſtance from the center of gravity. Hence to overcome ſmall obſtacles, the diſtance of the center of gravity from the

the great wheels : its diſtance from the ſmall :: the ſquare root of the radius of the ſmall wheel : the ſquare root of the radius of the large wheel. Now I find that the radii of the wheels of a common waggon are about 5 ft. 8 in. and 4 ft. 8 in. and the diſtance of the wheels, when narrow, about 6 ft. 6 in.; hence the center of gravity of the load of a waggon ought to be about 3,6 in. nearer to the higher than to the lower wheels. For a broad wheel waggon, where the diſtance of the wheels is about 7 ft. 10 in. the center of gravity ought to be about 4,2 in. nearer to the higher than to the lower.

It appears alſo that when W and x are given and x is very ſmall, P varies inverſely as the ſquare root of the radius of the wheel. Hence the advantage of a wheel to overcome a ſmall obſtacle varies as the ſquare root of the radius of the wheel. This reſiſtance of the obſtacle cauſes the wheel to turn, but this reſiſtance is not friction; for friction ariſes from the rubbing of the parts of one body againſt thoſe of another, whereas here the wheel only turns upon a point; the friction therefore only takes place at the axle, where the parts rub one againſt another. There is therefore no friction at the ground, unleſs when the wheels ſlide, which is the caſe when they are chained together, which is frequently done to prevent them from running too faſt down a hill.

Upon ſand.

107. It requires a leſs force to draw a narrow than a broad wheel carriage upon ſand.

The diſadvantage of the broad wheels ſeem to ariſe from their driving the ſand before them.

108. If two wheels be high and two low, it requires a greater force to draw the carriage than when all the wheels are high.

109. If all the wheels be low, it requires a greater force to draw the carriage than in the laſt caſe.

In all theſe caſes it requires a leſs force to draw the carriage when loaded behind than before.

HYDROSTATICS.

DEFINITION.

1. A FLUID is a body whofe parts are put in motion one among another by any force impreffed; and which, when the impreffed force is removed, reftores itfelf to its former ftate.

All fluids may be divided into elaftic and non-elaftic; elaftic comprehend the different kinds of airs, and non-elaftic the different kinds of liquids. As many bodies, by cold, from liquids become folids, fuch bodies might be defined to be liquids fo long as their furfaces, when difturbed, will reftore themfelves to an horizontal pofition. The definition fuppofes a partial preffure; for if the fluid be incompreffible, under an equal general preffure, none of the parts will move. From the eafe with which the parts of a fluid are moved, it is fuppofed to be conftituted of particles round and very fmooth.

2. The fpecific gravity, and alfo the denfity of a body, is in proportion to its quantity of matter or weight, when the magnitude is given.

ON THE PRESSURE OF FLUIDS.

1. A fluid weighs as much in a fluid of the fame kind, as it does out of the fluid.

2. The furface of every fluid at reft is horizontal.

3. If the denfity of a fluid be uniform, the preffure at any depth is in proportion to the depth.

4. Fluids prefs equally in all directions.

Hence the lateral preffure of a fluid is equal to the perpendicular preffure. This is one of the moft extraordinary properties of fluids, and can be conceived to arife only from the extreme facility with which the component particles move amongft each other. It will be difficult to conceive how this is poffible to happen if we fuppofe the particles to be in contact; they are therefore probably kept at a diftance from each

each other by a repulſive power, which power in water, admitting the truth of the experiments related, muſt be greater than any human power that can be applied. This is one remarkable difference between ſolids and fluids, as the former preſs only downwards.

This and the laſt prop. accounts for what is called the hydroſtatical paradox, by which very great weights may be balanced by a very ſmall weight of water without its acting at any mechanical advantage.

5. If two fluids of the ſame kind communicate, their ſurfaces will reſt in the ſame horizontal plane.

The *Romans* do not appear to have known this property of fluids, otherwiſe they would not have been at the expence of conveying their water from one place to another over arches built one upon another.

6. If a fluid preſs againſt any ſurface, the preſſure varies as its area multiplied into the depth of its center of gravity, the preſſure at every point being eſtimated in a direction perpendicular to the ſurface at that point.

The preſſures of different fluids will vary as above, and as their denſities conjointly.

7. The preſſure of a fluid againſt any ſurface is equal to the weight of a cylinder of that fluid, whoſe baſe is equal to the area of the ſurface and altitude equal to the depth of its center of gravity.

DEFINITION.

The center of preſſure of a ſurface is that point to which if a force equal to the whole preſſure were applied, but in a contrary direction, it would keep the ſurface at reſt.

8. If a plane ſurface which is preſſed by a fluid be produced to the ſurface of the fluid, and their common interſection be made the axis of ſuſpenſion, the center of oſcillation will be the center of preſſure.

9. If two different fluids communicate, they will

will ſtand at altitudes from the plane where they meet, which are inverſely as their ſpecific gravities or denſities.

If mercury and water thus communicate, the altitude of the latter will be about 14 times that of the former.

ON THE SPECIFIC GRAVITIES, OR DENSITIES OF BODIES.

10. The weight W of a body varies as its magnitude M and ſpecific gravity S conjointly.

A cubic root of rain water weighs 1000 ounces avoirdupoiſe; call the ſpecific gravity of this s, and then $W : 1000 :: M \times S : 1 \text{ ft.} \times s$, hence if we aſſume $s = 1000$ as a ſtandard to compare the ſpecific gravities with, we have $W = M \times S$, where M is the magnitude in feet. To reduce it to the meaſure in cubic inches we have ,1728 : 1 :: 1000 : ,5787 the weight of a cubic inch of rain; hence $W = ,5787\, MS$, where M is the magnitude in cubic inches. Now a troy ounce : an avoirdupoiſe ounce :: 480 : 437,5, for an avoirdupoiſe ounce contains 437,5 grains troy. If we reduce W to troy weight, which is moſt commonly uſed, we ſhall have $W = ,52746\, MS$ troy ounces $= 253,18\, MS$ grains. Hence if W and S be given, we know M the magnitude in cubic inches. By this method Mr. Atwood, in his Analyſis, conſtructed the following table to find the capacity of an irregular veſſel.

Let the veſſel be filled with water, the weight of which is A ounces, then ,52746 : A :: 1 : the capacity in cubic inches. Hence

oz.	cub.in.	oz.	cub.in.	oz.	cub.in.
1	= 1,8959	4	= 7,5835	7	= 13,2712
2	= 3,7918	5	= 9,4794	8	= 15,1671
3	= 5,6877	6	= 11,3753	9	= 17,0630

Hence we have a very accurate method of determining the diameter d of any ſphere whoſe weight is w and ſpecific gravity s, that of water being unity. For the ſolid content of a ſphere whoſe diameter is 1 is ,5236; hence 1 : ,5236 :: 253,18 grains : 132,428 grains the weight of a ſphere of water whoſe diameter is 1; hence, as the weights of ſpheres are as their ſpecific gravities and cubes of their diameters conjointly, we have $132,428\, sd^3 = w$, conſequently $d = \left.\frac{w}{s}\right|^{\frac{1}{3}} \times ,19612$, which is the rule given by Mr. Atwood.

11. If a body ſwim on a fluid, it will not reſt till its center of gravity is in a vertical line with the center of gravity of the water diſplaced.

For

For the body is ſupported by a force under it equal to the reaction of the fluid diſplaced, and the effect of the force of this fluid and of the body ſupported being the ſame as the effect of each concentrated in its center of gravity, their centers of gravity muſt be in the ſame vertical line, otherwiſe the body will not be ſupported.

12. If a body ſwim on a fluid, it diſplaces as much fluid as is equal in weight to the body; and the part immerſed : the whole body :: the ſpecific gravity of the body : that of the fluid.

The HYDROMETER is an inſtrument for finding the ſpecific gravities of fluids, and is conſtructed upon the principle of this propoſition, by meaſuring how far it ſinks in different fluids, and the parts immerſed are inverſely as the ſpecific gravities of the fluids. It is uſually a braſs ſtem with a bubble at the bottom into which ſomething heavy is put to make it ſink and keep the ſtem, which is graduated, upright, in order to ſhew how much it ſinks in different fluids; and by knowing the weight of the whole inſtrument and of any part of the ſtem, you determine their ſpecific gravities. This inſtrument is alſo made uſe of to find whether a liquor is above or below proof, by obſerving whether it ſtands above or below that point upon the ſtem which is proof. Many improvements have been made to this inſtrument, of which thoſe lately made by Mr. RAMSDEN ſeem to be the moſt important.

In this propoſition we neglect the effect of the air upon the part without the fluid, which is conſidered in prop. 17.

13. The weight which a body loſes when wholly immerſed in a fluid is equal to the weight of an equal bulk of the fluid.

14. The weight which a body loſes in a fluid : its whole weight :: the ſpecific gravity of the fluid : that of the body.

If the body which is weighed in a fluid be wood, it ſhould firſt be well rubbed over with greaſe, or be varniſhed, to prevent its imbibing any of the fluid.

When we ſay a body loſes part of its weight, we do not mean that it gravitates leſs than it did before, or that its real weight is leſs, but that it is partly ſupported by the reaction of the fluid upwards againſt its under ſurface, and therefore it requires a leſs weight to ſupport it; the weight thus ſaid to be loſt is communicated to the fluid, for the ſum of the weights of the body and fluid is juſt the ſame when the body is in the fluid as when out. By this propoſition the ſpecific gravity of a ſolid and fluid are compared.

Cor. 1. Hence if different bodies be weighed in the ſame fluid, their ſpecific gravities will be as their whole weights directly and the weights loſt inverſely.

If the body to be examined conſiſt of ſmall fragments, they may be put into a ſmall bucket and weighed, and then if from the weight of the bucket and body in the fluid we ſubtract the weight of the bucket in the fluid, there remains the weight of the body in the fluid.

Cor. 2. If the ſame body be weighed in different fluids, their ſpecific gravities will be as the weights loſt.

The body for this purpoſe ſhould not be wood, as that would imbibe the fluids; a ſolid glaſs bulb is the moſt proper.

Cor. 3. Hence if two bodies of different magnitudes balance each other in any fluid, the greater will preponderate in a lighter fluid, and the leſs in an heavier.

15. If Q be a body lighter than the fluid, connect it with another P which is heavier ſo that together they may ſink, and let the weight of P in the fluid be a, and the weight of $P+Q$ in the fluid be b, and let d be the weight of Q out of the fluid; then the ſpecific gravity of Q : the ſpecific gravity of the fluid :: $d : a-b+d$.

16. A body immerſed in a fluid, aſcends or deſcends with a force equal to the difference between its own weight and the weight of an equal bulk of the fluid.

17. If a lighter fluid reſt upon an heavier, and their ſpecific gravities be as $a : b$, and a body whoſe ſpecific gravity is c reſt with one part P in the upper fluid and the other part Q in the lower, then $P : Q :: b-c : c-a$.

Cor. Hence $Q : P+Q :: c-a : b-a$; and if a be ſo ſmall that it may be neglected, $Q : P+Q :: c : b$, as in prop. 12.

18. If a and b be the ſpecific gravities of two fluids to be mixed together, P and Q their magnitudes, and c the ſpecific gravity of the compound, then $P : Q :: b-c : c-a$.

It

It is here ſuppoſed that the magnitude of the two fluids when mixed is equal to the ſum of the two magnitudes when ſeparate. But it very often happens that the magnitude of the mixture is leſs than this ſum, owing, probably, partly to the conſtituent particles of the different fluids being of different magnitudes, and partly to their chemical attraction. This is called a penetration of dimenſions. If water and oil of vitriol be mixed together, the magnitude of the mixture is leſs than the ſum of the two magnitudes when ſeparate.

19. The aſcent of a light body in an heavier fluid ariſes from the preſſure of the fluid upwards againſt its under ſurface.

For if the body be placed upon the bottom of the veſſel, and ſo cloſely fitted to it that no part of the fluid can get under, it will reſt; but if it be lifted up ſo that the fluid gets under it, it immediately riſes.

20. If a plate of braſs be cloſely fitted to the bottom of a glaſs veſſel of the ſame ſize, and then immerſed in a fluid, when it is ſunk to about 8 times the depth of its thickneſs, it will then be ſupported by the fluid under it.

ON THE TIME OF EMPTYING VESSELS, AND ON SPOUTING FLUIDS.

21. If a fluid ſpout from the bottom or ſide of a veſſel, at a ſmall diſtance from the orifice the ſtream is contracted, and when the orifice is ſmall, the diameter of the ſmalleſt part : the diameter of the orifice :: 21 : 25, as determined by Sir. I. NEWTON.

When a fluid ſpouts from a veſſel, the water from all the ſides ruſhing towards the orifice is the cauſe of the contraction of the ſtream, called the *vena contracta*. Now the area of the orifice : the area of the ſmalleſt ſection of the ſtream :: $25^2 : 21^2$ which is very nearly as $\sqrt{2}$: 1; hence, as the velocity is inverſely as the area of the ſection, the velocity at the vena contracta : the velocity at the orifice :: $\sqrt{2} : 1$. Now from the quantity of water running out in a given time, and the area of the vena contracta, Sir I. NEWTON found that the velocity there was that which a body acquires in falling down the altitude of the fluid above the orifice; hence the velocity at the orifice being leſs in the

ratio of $\sqrt{2} : 1$, muſt be that which is acquired in falling down half the altitude. We muſt therefore diſtinguiſh between the velocity at the orifice and at the *vena contracta*, and in the doctrine of ſpouting fluids, it is the latter velocity which we muſt conſider, and aſſume the point of projection from that point. This is true only upon ſuppoſition that the orifice is very ſmall; if not, the velocity there is leſs than that acquired in falling down half the height. It appears however from ſome experiments which I have made, that the velocity of the effluent water is in no *conſtant* ratio of any part of the depth; when the orifice is very ſmall it appears that the velocity there is very nearly that which is acquired in falling down half the depth; but as the ſurface of the fluid deſcends the velocity is greater than that acquired through half the depth. See PARKINSON's Hydroſtaticks, p. 83.

22. If a veſſel empty itſelf through an orifice at the bottom, and the area of the ſection parallel to the bottom continue the ſame, the velocity of the ſurface of the fluid is uniformly retarded.

23. The velocity of a fluid through different ſections of the ſame tube or veſſel varies inverſely as the area of the ſection.

If $a =$ the ſurface of a fluid running out of an orifice o at the bottom of a veſſel, $x =$ the depth of the fluid at that time, $t =$ the time the fluid has been running; then $\dot{t} = \frac{a}{o} \times \frac{-\dot{x}}{\sqrt{2sx}}$, where $s = 16\frac{1}{12}$ feet.

Cor. 1. If the veſſel be cylindrical or priſmatic, and $h =$ the altitude, then $t = \frac{a}{o} \times \sqrt{\frac{2h}{s}}$ the time of emptying.

Cor. 2. The time of the deſcent of the fluid in the ſame veſſel through any ſpace $z = \frac{a}{o\sqrt{s}} \times \overline{\sqrt{2h} - \sqrt{2h - 2z}}$.

If the depth of a cylindrical veſſel be 12 in. and the diameter of the orifice at the bottom be 0,3 inches, the time of emptying by theory = 35′, which agrees very nearly with experiment.

24. If a cylindrical veſſel be kept conſtantly filled with a fluid, twice the quantity which the veſſel contains will run out in the time it would have emptied itſelf.

This appears by mechanics prop. 60. becauſe the ſurface of the fluid, when the veſſel empties itſelf, is uniformly retarded.

In

In all the propoſitions reſpecting the times in which veſſels empty themſelves, the orifice is ſuppoſed to be very ſmall in reſpect to the bottom of the veſſel, in which caſe our theory and experiments agree very nearly; otherwiſe they do not. This agreement of the theory with experiment, proves the velocity at the orifice, whoſe magnitude is under the above reſtriction, to be that which a body acquires in falling down half the depth of the fluid, the theory being founded upon that principle. As the orifice increaſes, the velocity is that which is acquired in falling down leſs than half the depth.

25. If a cylindrical veſſel 12 inches high have a circular orifice at the bottom of 0,3 in. diameter, it will empty itſelf in about 35″; but if a pipe of the ſame length and diameter be fixed in the bottom, and the fluid flow through it, the veſſel will be emptied in about 18″, whether, when you fill the veſſel, you ſtop the tube at the top or the bottom; but if the pipe increaſe downwards, and the diameter of the lower end be 0,42 in. the time of emptying will be about 13″.

26. If a cylindrical veſſel 18 in. deep have a circular hole at the bottom, and at the ſide cloſe to the bottom, of 0,45 in. diameter; alſo if a cylindrical pipe 13,7 in. long and of the ſame diameter be fixed in the ſide by the bottom in an horizontal poſition; in all theſe caſes the time of emptying is about 25″; but if the pipe increaſe in its diameter, ſo that the diameter of the other end be 0,65 in. the time of emptying is about 16″.

27. If a veſſel be filled with a fluid, and from any point it be directed to ſpout upwards, it riſes nearly to the height of the ſurface of the fluid in the veſſel.

Two cauſes prevent it from riſing quite ſo high, one is the reſiſtance of the air, and the other is the falling back of the fluid. Hence, in

confequence of this latter obftruction, the fluid will rife higher if the direction deviate a little from the perpendicular. In great velocities the refiftance is fo great, that GRAVESANDE fays the greateft height to which water can thus be projected is not above 100 feet.

28. The diftance to which a fluid fpouts from the fide of a veffel upon an horizontal plane, is equal to twice the correfponding ordinate of a circular arc whofe diameter is equal to the diftance of the furface of the fluid from the plane.

From the diftance to which a fluid fpouts from the fide of a veffel, it appears that the velocity with which it fpouts muft be that which is acquired in falling down the whole depth of the fluid above the orifice. But it has been fhown that the velocity through the fection of the orifice is that which is acquired in falling through half the depth of the fluid. Hence as foon as the fluid gets out of the orifice, it acquires an increafe of velocity in the ratio of $\sqrt{2} : 1$. The preffure of the fluid in the veffel muft therefore continue to act upon the effluent fluid for a fmall diftance after it has paffed the fection of the orifice.

ON THE ATTRACTION BETWEEN SOLIDS AND FLUIDS.

29. If two glafs planes, forming a very fmall angle with each other, be dipped in water, the water will rife between them, and the height will be greater the nearer it is to the concourfe of the planes. Mercury will in like manner fink.

The water is fuppofed to afcend by the attraction of the glafs which lies contiguous to the furface of the fluid. The mercury defcends, being more attracted by itfelf than by the glafs. The upper furface of the fluid will form the common hyperbola, one of whofe afymptotes is the common interfection of the planes, and the other lies in the furface of the fluid in the veffel. Where the diftance is about $\frac{1}{100}$ of an inch, the water rifes about an inch. If the planes be parallel, the height is every where the fame. Mr. HAUKSBEE tried the fame with *marble* and *brafs* planes, and found that the water rofe between them.

30. If very fmall glafs tubes be dipped in water, the water will ftand in them, above the level of that in the veffel, at altitudes which are inverfely as their diameters.

Hence

Hence the product of the diameter and altitude is a constant quantity, which quantity $=,053$ of an inch. To measure the diameter of a capillary tube, put in some mercury whose weight is w, and let it occupy a length of the tube l; then if 13,6 be the specific gravity of mercury (which it is when purest) when that of water is 1, the diameter $= \sqrt{\frac{w}{l}} \times ,01923$. For if $d =$ diameter, the content of the mercury $= d^2 l \times ,7854$, and as one cubic inch of mercury weighs 3443 grains, we have $1 : 3443 :: d^2 l \times ,7854 : w$, hence $d = \sqrt{\frac{w}{l}} \times ,01923$. This rule is given by Mr. ATWOOD in his Analysis, p. 3.

It has been generally supposed that the water is suspended by the attraction of a small annular surface on the inside of the tube, contiguous to the surface of the water. But Dr. HAMILTON (Lect. 2. p. 47.) supposes it to arise from the attraction of the annulus lying just within the lower orifice of the tube. Mr. PARKINSON however, in his System of Hydrostatics, thinks neither of these suppositions true, but supposes that the fluid is sustained by the immediate attraction of the glass. As the fluid will rise in an exhausted receiver, it cannot in any way be owing to the air.

31. If a be the altitude at which water will stand in a capillary tube whose diameter is d, then if another tube whose lower part is larger but whose diameter is d at the altitude a has its air drawn out, the water will rise and be suspended at that altitude.

In this case it is supposed that the attraction of the annular surface contiguous to the surface of the water supports all the fluid immediately under that part of the tube, and the other part is supported by the pressure of the air on the surface of the fluid without the tube. That this is true appears from hence, that the experiment will not succeed in a well exhausted receiver. Dr. JURIN tried it in an exhausted receiver and says that it did succeed; this therefore must have happened from his air pump not exhausting sufficiently. Dr. HOOK makes the greatest altitude in the finest capillary tube about 21 inches.

32. If a glass tube be filled with ashes rammed very close, and the lower end be dipped in water; the water will gradually rise up through the ashes.

Mr. HAUKSBEE took a tube 32 inches long, and found that the water ascended to the top in 130 hours. He then asks, "does not this arise from the same cause as in small tubes, or between two planes? and

and do not the particles of this matter, by their little hollows and intervals, form a congeries of minute ſlender pipes, or ſurfaces near each other, ſo that the liquor riſes by one and the ſame cauſe?" It riſes faſter in vacuo than in the open air, becauſe in the latter caſe it has all the air to force out. Water will alſo riſe in ſalt, ſugar, &c. in the ſame manner. The aſcent of juices in vegetables, and the various ſecretions of fluids through the glands of animals, ariſe from the ſame cauſe, the power of attraction.

33. If the lower of two glaſs planes forming a very ſmall angle with each other be parallel to the horizon, and their ſurface be moiſtened with oil of turpentine, oranges, &c. a drop of oil placed between them will move towards their concourſe.

ON THE RESISTANCE OF FLUIDS.

34. If a body move in a reſiſting medium, the reſiſtance, within certain limits of the velocity, varies very nearly as the ſquare of the velocity.

The reſiſtance ariſes from three cauſes, the inertia, the tenacity, and the friction of the fluid. The latter cauſe is inconſiderable, and the ſecond is, in moſt fluids, but very ſmall when compared with the inertia. The reſiſtance ariſing from the tenacity will be diminiſhed as the velocity increaſes, but that ariſing from the inertia will be, within certain limits, as the ſquare of the velocity. In very ſwift motions, the reſiſtance of the air increaſes in a greater ratio, for the reaſon explained in prop. 90. of mechanics; and in other fluids the ſame conſequence would follow, for the ſame reaſon, for projected bodies. But when bodies deſcend in fluids ſuch as water, the reſiſtance is always very nearly as the ſquare of the velocity, becauſe the body never can acquire a velocity beyond a certain limit. The greater the velocity is the leſs will be the preſſure againſt the back part of the body, and this variation of preſſure will cauſe a deviation in the law of reſiſtance. Air, water and mercury are called perfect fluids, not having any ſenſible tenacity or friction. This doctrine of reſiſtance was eſtabliſhed by Sir I. Newton, by a variety of experiments. See the Principia, Vol. II. prop. 31. Scholium.

35. When plane bodies move in reſiſting mediums, the reſiſtance varies as their areas, ſquares of their velocities and denſities of the mediums conjointly.

36. If

36. If a plane body move obliquely in a reſiſting medium, and the force of the oblique ſtroke : the force of the direct ſtroke :: the ſine of the angle at which the plane ſtrikes the fluid : radius, the reſiſtance perpendicular to the plane varies, cæteris paribus, as the ſquare of the ſine of the angle at which the plane ſtrikes the fluid.

The principle upon which this propoſition is founded is not true when the body moves in air, as will be ſhown by experiment. I have not yet had an opportunity of examining whether it be true for water. The two next propoſitions ſuppoſe the truth of the ſame principle, and therefore are not true for air.

37. The reſiſtance of the ſame plane in the direction of its motion varies as the cube of the ſine of the ſame angle.

38. The reſiſtance of the ſame plane in a direction perpendicular to its motion varies as the ſquare of the ſine of the ſame angle into its coſine.

Hence if we ſuppoſe the plane to be at reſt and the fluid to move againſt it, the effect of the fluid to move the plane in a direction perpendicular to the motion of the fluid will vary in the ſame ratio. This, if the above principle were true, would be the effect of the wind to put the ſails of a mill in motion. After they have begun their motion, the effect will depend upon the relative velocity of the ſails and wind; the angle alſo at which the wind acts upon the ſail will vary with the velocity of the ſail.

39. If a plane body move in water in a direction which is perpendicular to its ſurface, it is reſiſted by a force equal to the weight of a column of the fluid whoſe baſe is equal to that ſurface, and height equal to that through which a body muſt fall to acquire the velocity of the body.

See PARKINSON's Hydroſtatics, p. 173. and ATWOOD on Rectilinear and Rotatory Motion, p. 124. This prop. is capable of a very ſatisfactory proof by experiment.

40. If

40. If a ſphere and end of a cylinder of the ſame diameter moving in the direction of its axis, move in a fluid with equal velocities, the reſiſtance of the cylinder will, by theory, be double that of the globe.

In the demonſtration of this prop. the principle in prop. 37. is aſſumed as true. The prop. therefore is not true if the globe move in air.

DEFINITION.

If a plane body revolve in a reſiſting medium about an axis by means of a weight hanging from it, that point into which if the whole plane were collected it would ſuffer the ſame reſiſtance, I call the *center of reſiſtance.*

41. If a be the area of the plane, and $\dot{a}$ the fluxion of the area at the diſtance x from the axis, then the diſtance d of the center of reſiſtance from the axis $= \sqrt[3]{\frac{\text{flu.}\, x^3\dot{a}}{a}}$.

For the effect of the reſiſtance of $\dot{a}$ to oppoſe the weight muſt vary as the reſiſtance into its diſtance from the axis, or (becauſe the reſiſtance varies as the ſquare of the velocity, or as the ſquare of the diſtance from the axis,) as $x^3\dot{a}$, and therefore the whole reſiſtance is as the fluent of $x^3\dot{a}$. For the ſame reaſon, the effect of the reſiſtance of a at the diſtance d is as d^3a; hence $d^3a = \text{flu.}\, x^3\dot{a}$, therefore $d = \sqrt[3]{\frac{\text{flu.}\, x^3\dot{a}}{a}}$.

If the body be a parallelogram, two of whoſe ſides are parallel to the axis and at the diſtances m and n, m being the leaſt diſtance, then $d = \sqrt[3]{\frac{n^4 - m^4}{4n - 4m}} = \sqrt[3]{\frac{\overline{n^2 + m^2} \times \overline{n + m}}{4}}$.

42. If a plane body revolve about an axis by a moving force acting thereon, firſt with its edge forward and then with its plane ſide, the difference of the accelerative forces which will preſerve the motion of the body in each caſe uniform with the ſame velocity, diminiſhed in the ratio of the diſtance of the center of reſiſtance from the center of the

the axis to the radius of the axis, gives the abſolute reſiſtance of a plane equal to the body moving with the velocity with which the center of reſiſtance moves.

The machine conſtructed for this purpoſe is an horizontal axis, at one end of which there are four arms, like the ſails of a mill, and at each extremity a plane is fixed. Two lines are wound round the axis together, and one leaves the axis above and the other below it, and go in oppoſite directions horizontally and paſs over pullies, and equal weights are hung on at each end to give motion to the axis. By this means, all preſſure upon the axis from the weights is avoided, ſo that whatever change it may be neceſſary to make in the weights to give the ſame velocity to the planes going flat and edge ways, there is not more preſſure upon the axis, and conſequently not more friction. The difference of the weights therefore can only ariſe from the reſiſtance of the planes moving edge and flat ways; and a weight at the center of reſiſtance equivalent to that difference, muſt, from the common property of the lever, be to that difference as the radius of the axis to the diſtance of that center from the center of the axis. To render the reſiſtance ſtill greater, four arms might be put upon the other end of the axis. In my machine the radius of the axis is 0,199 in. the length of the arms 30 in. each plane is a ſquare whoſe ſide is 4 in. one ſide of which is parallel to the arm, and the center of the ſquares is 32 in. from the axis. Hence the diſtance of the center of reſiſtance from the axis $=$ 32,044 in. The radius of the axis was determined in the following manner. A fine ſilk thread 36 in. long was all wound round the axis ſo that the whorls all touched each other, and the number of revolutions and parts of a revolution were obſerved; then dividing 36 in. by the number of revolutions and parts of a revolution, it gave the circumference of the axis, from which the radius was determined. By this method the radius of any cylinder may be found to a very great degree of accuracy, and the longer the ſtring the greater the accuracy. As the ſtring by winding runs along the axis, it does not go in a plane perpendicular to the axis, and therefore there will be a ſmall inaccuracy from that, but it is ſo ſmall that there can ſcarce be a caſe where it may be neceſſary to conſider it. If there ſhould, it may be corrected thus. From the ſquare of the length of the ſtring ſubtract the ſquare of the ſpace it takes upon the axis, and the ſquare root of the difference is the quantity to be divided by the number of revolutions in order to get the circumference. The mean of 7 experiments gave the reſiſtance $=$ 0,0462 oz. troy to 64 ſquare inches ſtriking the air perpendicularly with a velocity of 2 feet in a ſecond

43. If a plane body ſtrike the air obliquely, the effect of the direct ſtroke is not to that of the oblique, as radius to the ſine of the angle of incidence.

This

This was discovered by inclining the planes at different angles, and observing what weights would give them the same uniform velocity. The angles at which I first tried the experiment gave the force of the stroke greater in proportion than the sine of inclination. I communicated to Dr. HUTTON what I had done upon this subject, knowing that he had made experiments on resistances of the air upon a very extensive scale; when he was so obliging as to inform me, that having made experiments of the same kind at all angles of obliquity from 0° to 90°, (which I had not then done,) he found that in one part of the quadrant the force was greater and in the other part less, than in proportion to the sine of inclination, and that the variation followed a very extraordinary law. He finds the resistance of a globe $2\frac{2}{5}$ greater than the resistance of a cylinder, moving in the direction of its axis with the same velocity. It is hoped that he will soon communicate to the public his valuable experiments.

44. The resistance of the air to plane bodies varies, cæteris paribus, as the square of the velocity.

For it requires four times the above mentioned difference of weights to be hung from the axis to give the planes twice the velocity.

ON THE MOTION OF BODIES IN FLUIDS.

45. If a sphere whose weight is w and diameter d move in a resisting medium with a velocity which a body would acquire in falling in a vacuum through the space h, and the density of the medium : the density of the body :: 1 : b; then the resistance of the sphere $=\frac{3wh}{4db}=\frac{pd^2h}{8}$, where $p=$ 3,14159 &c.

See ATWOOD's Treatise on Rectilinear and Rotatory Motion, p. 130; also PARKINSON's Hydrostatics, p. 177.

46. The gravity of the same sphere in the fluid, being equal to its own weight diminished by the weight lost, is equal to $w-\frac{w}{b}$, the gravity of the body itself in vacuo being w.

47. The

47. The force with which the same sphere descends in the fluid, being equal to its gravity in the fluid diminished by the resistance, will be $w - \frac{w}{b} - \frac{3wb}{4db}$.

48. If $v =$ the velocity acquired by the same sphere in descending from rest through the space x, then by mechanics prop. 72. $v\dot{v} = 2mF\dot{x} = \overline{w - \frac{w}{b} - \frac{3wb}{4db}} \times 2m\dot{x}$.

If $e = 2,71828$ the number whose hyp. log. is 1, then if the fluent be taken and properly corrected, we have $v = \sqrt{\frac{16dm}{3}} \times \overline{b-1} \times \sqrt{1 - e^{\frac{-3x}{4bd}}}$; see PARKINSON's Hydrostatics, p. 186. and ATWOOD on Rectilinear and Rotatory Motion, p. 140.

If x be increased sine limite, $e^{\frac{-3x}{4bd}} = o$; hence $v = \sqrt{\frac{16dm}{3} \times \overline{b-1}}$ the limit of the velocity which the body can acquire, but which it can never attain. Or if d be very small when compared with x, $e^{\frac{-3x}{4bd}}$ will be very nearly $= o$; and hence if d be very small, x may be small and $e^{\frac{-3x}{4bd}}$ will $= o$ very nearly. Hence, as Mr. ATWOOD observes, very small bodies descending in a fluid very soon acquire, as to sense, their greatest velocity, and then they appear to descend uniformly. He computes that if $x = 16d$, the velocity, after the sphere has descended 16 diameters, will be within less than $\frac{1}{800}$ part of the greatest velocity. Hence when a metal is dissolved in a menstruum, the particles being extremely small, will descend with so very small a velocity that they will for a long time, as to sense, appear quiescent. Mr. MACKBRIDE supposes that fixed air is the cementing principle of bodies; and Mr. ATWOOD supposes that when a body is dissolved in a menstruum, the fixed air escaping carries up the disunited parts of the body; and when the medium is once filled with the particles, they will, as is shown above, remain suspended for a very long time. The dissolution of the body is supposed to arise from hence, that the particles of the medium attract the particles of the body with a greater force than the particles of the body attract each other.

49. If t be the time in which the ſphere in the laſt prop. deſcends through the ſpace x, then $t =$

$$\sqrt{\frac{b^2d}{3m \times \overline{b-1}}} \times \text{hyp. log.} \frac{1+\sqrt{1-e^{\frac{-3x}{4bd}}}}{1-\sqrt{1-e^{\frac{-3x}{4bd}}}}.$$

See ATWOOD on Rectilinear and Rotatory Motion, p. 150. and PARKINSON's Hydroſtatics, p. 189.

This is upon ſuppoſition that the reſiſtance varies as the ſquare of the velocity; and the experiments compared with the theory agree ſufficiently to eſtabliſh the truth of the hypotheſis. A ſmall difference muſt neceſſarily ariſe, granting the ſuppoſition to be true, from the unavoidable errors in conſtructing the experiments.

In our experiments $d = 2{,}0833$ in. $b = 1{,}01014$, $x = 55{,}5$ in.; hence $t = 12''$, 75 the time by theory. The time by experiment is about $13''$.

The diameter was determined by the rule in prop. 10. by loading the ſphere till it was of the ſame ſpecific gravity as water, in which caſe its weight was 1198,814 grains; hence $s = 1$, and its diameter $=$ $\overline{1198{,}814}^{\frac{1}{3}} \times {,}19612 = 2{,}0833$ inches.

50. As a body deſcends in a fluid, it continually adds more weight to the fluid until it has acquired its greateſt velocity, at which time the weight added to the fluid is juſt the ſame as if the body were laid at the bottom of the veſſel.

For as the velocity of the body keeps increaſing, the action of the body upon the fluid will keep increaſing; and when the body has acquired its greateſt velocity, the reſiſtance being equal to the weight of the body in the fluid, the body acts againſt the fluid with its relative weight juſt as it would act againſt the bottom of the veſſel if it were laid upon it.

ON ELASTIC FLUIDS, AND THE DENSITY OF THE AIR AT DIFFERENT ALTITUDES.

51. If the particles of an elaſtic fluid repel each other with forces varying inverſely as the n^{th}. power of their diſtance, then if r repreſent the diſtance of the

the particles, d the denſity and c the compreſſive force, c varies as $d^{\frac{n+2}{3}}$.

Cor. It appears by experiment, that in the common atmoſpheric air the compreſſive force varies as the denſity. Hence $\frac{n+2}{3}=1$, conſequently $n=1$, and therefore the particles of air repel each other with forces which vary inverſely as their diſtance.

As the compreſſive force is equal to the elaſtic force of the air, action and reaction being equal and contrary, the elaſtic force muſt vary as the denſity.

52. If we ſuppoſe the force of gravity to vary as the n^{th}. power of the diſtance from the earth's center, r the radius of the earth, x any diſtance from the center, and v the correſponding denſity, that at the earth's ſurface being unity, and d be the height of an homogeneous atmoſphere; then $d \times$ hyp. log. $v=\frac{r}{n+1}-\frac{x^{n+1}}{\overline{n+1}\times r^n}$.

Cor. 1. If we ſuppoſe the force of gravity to vary inverſely as the ſquare of the diſtance, then $n=-2$; hence $d\times$ hyp. log. $v=\frac{r^2}{x}-r$. Hence if x be taken in muſical progreſſion, $\frac{r^2}{x}$, and conſequently $\frac{r^2}{x}-r$, will be in arithmetical progreſſion, therefore the hyp. log. v will be in arithmetical progreſſion, and hence v will decreaſe in geometrical progreſſion.

Cor. 2. If we ſuppoſe gravity to be conſtant, then $n=0$, therefore $d\times$ hyp. log. $v=r-x$; hence if x increaſe in arithmetic progreſſion, $r-x$, and conſequently hyp. log. v, will decreaſe in arithmetic progreſſion, and therefore v will decreaſe in geometrical progreſſion, conſequently the log. of the denſity decreaſes as the altitude increaſes.

From experiments on the denſity of the air at the bottom and top of hills, Mr. Cotes (Hydroſtatics, p. 103.) collected, that at the altitude of 7 miles the denſity was four times leſs than at the earth's ſurface, or $=\frac{1}{4}$; hence if $z=$ the diſtance in miles above the earth's ſurface, $7:\log.\frac{1}{4}::x:\log. v=\frac{x}{7}\times\log.\frac{1}{4}$, therefore $v=\overline{\frac{1}{4}}\big|^{\frac{x}{7}}$. Or to expreſs it in terms

of

of the rarity r, we have $7 : log. 4 :: x : log. r$, and hence $r = \overline{4}|^{\frac{x}{7}}$; alſo $x = 7 \times \frac{\log. r}{\log. 4} = 1{,}1626 \times \log. r$ miles. This rule ſuppoſes the denſity to be as the compreſſive force, which is not true unleſs the temperature remains the ſame; but as the temperature is found to be very different at the ſame time at different altitudes, the rule will require a correction according to the altitudes of the thermometers at the two places. Omitting however this correction, the denſity of the air at the altitude of 45 miles is found to be 7420 times leſs than at the earth's ſurface; and yet, from obſervations on the twilight, the rays of light are ſenſibly affected by the air at that altitude.

ON THE BAROMETER.

53. If a glaſs globe be exhauſted of air and balanced at one end of a beam, upon admitting the air the globe preponderates.

This experiment clearly proves that the air has weight; and from the weight neceſſary to balance the globe after the admiſſion, the weight of the air will be known. Mr. COTES found the denſity between 800 and 900, but nearer to 900, times leſs than water; and Mr. HAUKSBEE made it 885 times leſs, when the barometer ſtood at $29\frac{1}{2}$ inches. Hence, as a cubic inch of water weighs 253,18 grains troy, a cubic inch of air weighs 0,286 grains. If we take mercury to be 14 times heavier than water, the ſpecific gravity of air : that of mercury :: 1 : 8851 $\times$ 14 = 12390.

54. If a glaſs tube more than 31 inches long, hemetically ſealed at one end, be filled with mercury and then inverted and its end immerſed in a baſon of the ſame fluid, it will ſtand at an altitude above the ſurface of the mercury in the baſon between 28 and 31 inches.

As the mercury deſcends from the top of the tube it muſt leave a vacuum, and it remains ſuſpended by the preſſure of the air upon the ſurface of the mercury in the baſon, for if the air be taken off from the ſurface the mercury deſcends. This preſſure of the air was diſcovered by GALILEO. He found by experiment that water might be raiſed by the common pump to a certain height, and no higher; whereas, had nature abhorred a vacuum, as the philoſophers then thought, it might have been raiſed to any height. He conjectured therefore that it was owing to the air's gravitation. Afterwards his pupil TORRI-

CELLIUS considered, that if the pressure of the air would support a column of water about 35 feet high, it must suspend a column of mercury, whose density is about 14 times greater, about one fourteenth part of 35 feet; he according tried the experiment in the proposition and found that the mercury stood at the height which he expected. Thus he fully proved the air's pressure, and hence this is called the *Torricellian* experiment, and the vacuum which is left above is called the *Torricellian* vacuum. A tube thus filled and graduated from 28 to 31 inches is called a *Barometer*.

Hence we get the altitude of an homogeneous atmosphere; for by the last article when the mercury stood at an altitude of $29\frac{1}{2}$ inches, the density of the air was to that of mercury :: 1 : 12390; hence the altitude of an homogeneous atmosphere $= 12390 \times 29\frac{1}{2} = 365505$ in. $= 5{,}77$ miles. If, according to some experiments, we suppose that when the mercury in the barometer stands at 30 inches the density of the air is 850 times less than that of water, the altitude of an homogeneous atmosphere would be 5,6 miles.

55. If a barometer be placed under the receiver of an air pump, and the air be exhausted, the mercury will descend; and upon admitting the air it ascends again to the former height.

56. If a bottle be partly filled with mercury, and through the cork, made air tight, a glass tube open at both ends be put so that the lower end be immersed in the mercury, then if the whole be put under the receiver of an air pump, and the air be exhausted, the mercury will rise in the tube nearly to the height at which the barometer stands at that time, or to the height at which the mercury rises in the gage.

This arises from the elasticity of the air being as its compressive force; a very small quantity therefore of air by its elasticity produces the same effect as the weight of the atmosphere. The mercury does not rise exactly to the height, first, because you cannot exhaust all the air, and secondly, because as the mercury rises in the tube, the air in the bottle occupies a greater space, and therefore its density and elastic force is diminished and become less than that of the air in its natural state.

57. If

57. If a barometer having its lower end immerſed in a baſon of mercury be ſuſpended from the beam of a balance, it is found to weigh as much as when you invert it with the ſame quantity of mercury in it, and ſuſpend it by the other end.

It might at firſt be thought, that in the firſt poſition the weight required to balance the barometer would be only equal to the weight of the glaſs tube, the mercury within being ſupported by the preſſure of the air upon the mercury without the tube; but as there is a vacuum left at the top, there is nothing to counterbalance the preſſure of the air againſt the top on the outſide; the tube therefore has to ſupport a column of air having the ſame baſe as the baſe of the tube, which column of air is equal in weight to the mercury within the tube.

58. If a barometer be carried to an altitude of 54 feet, the mercury is obſerved to ſink about $\frac{1}{20}$ of an inch.

59. If a be the altitude of mercury in a barometer at the bottom of a mountain, and b the altitude at the top, then the altitude of the mountain $= 1,1626 \times \log. \frac{a}{b}$ miles.

For at the top of the mountain the denſity of the air muſt be $\frac{b}{a}$, that at the bottom being unity, or we may call the rarity $\frac{a}{b}$, which ſubſtitute for r in the note to prop. 52. and the altitude $= 1,1626 \times \log. \frac{a}{b}$. The difference of temperature is not here conſidered.

60. When the mercury ſtands in the barometer at the altitude of 30 inches, the preſſure of the air upon every ſquare inch is about $18\frac{1}{2}$ lb. troy, or a little more than 15 lb. avoirdupoiſe.

For a cubic inch of mercury weighs 3544 grains troy, therefore 30 cubic inches weigh about $18\frac{1}{2}$ lb. Hence if we take the ſurface of a middle ſize man to be $14\frac{1}{2}$ ſquare feet, when the air is lighteſt the preſ-

ſure

ſure on him is $13\frac{1}{5}$ tons, and when heavieſt $14\frac{3}{10}$ tons, the difference of which is 2464 lb. This difference of preſſure muſt greatly affect us in regard to the animal functions, and conſequently in reſpect to our health, more eſpecially when the change takes place in a ſhort time. The preſſure of air upon the whole ſurface of the earth = 12043468800000000000 lb.

61. If the tube of a barometer, being perfectly cylindrical, be partly filled with mercury before it is inverted, after inverſion the mercury will ſink below the ſtandard altitude, and the ſtandard altitude will be to the depreſſion below that altitude as the ſpace occupied by the air after inverſion to the ſpace occupied before.

It is here ſuppoſed that the elaſtic force is as the denſity, and as the elaſtic force is equal to the compreſſive force, they balancing each other, the compreſſive force is as the denſity. If therefore the truth of this prop. appear from experiment, it follows that the elaſtic and conſequently compreſſive force of the air muſt be as its denſity.

ON THE AIR PUMP.

62. If the capacity of the barrel of an air pump : the capacity of the receiver :: $b : r$, after every turn, the quantity of air extracted : the quantity before :: $b : b+r$.

Cor. 1. Hence the quantities taken away at any number of ſucceſſive turns form a geometric ſeries, conſequently the whole can never be exhauſted.

Cor. 2. Hence, dividendo, the quantity remaining after every turn : the quantity before :: $r : b + r$, conſequently the quantities which remain after any number of ſucceſſive turns will form a geometric ſeries.

63. After every turn, the denſity of the air is diminiſhed in the ratio of $b+r : r$; and hence after t turns, it is diminiſhed in the ratio of $\overline{b+r}|^{t} : r^{t}$.

64. The defects of the mercury in the gage from the ſtandard altitude, after any number of ſucceſ-

five turns, form a geometric series whose terms are in the ratio of $b+r:r$.

65. The altitudes of the mercury in the gage at the same time form a geometric series, the ratio of whose terms is $b+r:b$.

66. When the air is rarified n times, the number of turns $=\dfrac{\log. n}{\log. \overline{b+r} - \log. r}$.

67. Air is necessary for the production of sound.

68. Air is necessary for the propagation of sound.

69. A candle will not burn in vacuo.

70. If the pressure of the air be taken off from one side of a thin glass phial, the pressure of the air on the other side will break it.

The experiments with the air pump, showing the very extraordinary effects of the pressure of the air, are so numerous, that it would take up too much room to insert them all here.

ON THE CONDENSER.

71. After every descent of the piston, one barrel of air in its natural state is forced into the receiver.

72. If the capacity of the receiver : the capacity of the barrel as $r:b$, then after t descents of the piston, the density is increased in the ratio of $b:b+rt$.

73. After any number of successive descents, the density is increased in arithmetic progression.

74. If

74. If the gage tube lie horizontal, the ſpaces which the air occupies after any number of ſucceſſive deſcents of the priſton will decreaſe in muſical progreſſion.

75. A bell in condenſed air ſounds louder than in air in its natural ſtate.

The effect of condenſed air may be ſhown by a variety of experiments. Fire engines, air guns, artificial fountains, &c. act from this cauſe.

ON PUMPS AND SYPHONS.

76. Water in pumps is raiſed by the preſſure of the air upon the ſurface without.

A *common pump* is formed with two ſuckers, each valve of which opens upwards; the lower ſucker is fixed, and the upper moveable by the handle.

The lower ſucker muſt not be more than 36 feet above the ſurface of the water in the well. For the water riſes by the preſſure of the air upon the water without, in conſequence of a vacuum within; and in the moſt condenſed ſtate of the atmoſphere, the preſſure of the air is not greater than that of a column of water 36 feet high, having equal baſes. But if a ciſtern there receive the water, and in like manner a pump works in that, water may be raiſed to any height.

A *forcing pump* has two ſuckers, the upper of which is moveable without a valve, and the lower is fixed with a valve opening upwards.

Some forcing pumps act by the force of the upper ſucker upon the water, and ſome by condenſed air upon it.

77. If one leg of a ſyphon be put into a veſſel of water and the air be drawn out of the other leg, the water will flow out of the leg without, provided the end be lower than the ſurface of the water in the veſſel.

The water will continue to run till the ſurface of the fluid is level with the end of the ſyphon without, and then it will ſtop.

The water is made to flow through the ſyphon by the preſſure of the air upon the ſurface of the water in the veſſel, the air being drawn from the ſurface of the water in the ſyphon. When the legs become equal, the preſſure of the air againſt the water at the end of the ſyphon without being equal to the preſſure of the air on the ſurface of the

 water

water in the veſſel, and theſe having columns of water of the ſame altitude to ſupport muſt balance each other, and conſequently the fluid will then ceaſe to flow; whereas it flowed before from the ſuperior preſſure of the column without.

78. If the ſyphon be capillary, the water will not flow out, till the end without be further below the ſurface of the fluid in the veſſel than the height to which the fluid would riſe in the tube by capillary attraction.

ON THE THERMOMETER.

79. Fluids expand by being heated, and contract again as they grow cold.

Hence a fluid whoſe expanſion by heat is very ſenſible and uniform, and not ſubject to be frozen, is proper for the conſtruction of a thermometer.

80. The expanſion of mercury, linſeed oil and ſpirits of wine, is, as to ſenſe, proportional to the heat applied.

For let a thermometer conſtructed with theſe fluids be put into cold water, and then into water heated to any degree, and note the altitudes; put equal quantities of theſe two waters together, which will give a mean heat, and the fluid will ſtand at the mean altitude between the two before obſerved. This is found to be true, of whatever temperatures the two parts of water are. Mercury is the moſt proper of the three fluids, as it is capable of enduring the greateſt degree of heat or cold without boiling on congelation. The thermometer uſually conſiſts of a ſmall glaſs cylinder with a glaſs ball at the bottom, generally a globe, but it is better to make it flat, becauſe all the mercury in it will then be the ſooneſt affected by the variation of the air's temperature. The points where the fluid ſtands in the ſtem at *freezing* and in *boiling* water are uſually noted by obſervation, and then the whole ſcale is divided into equal parts and numbered. In FAHRENHEIT's thermometer, the freezing point is at 32, and boiling water at 212. According to this diviſion, mercury boils at 600, and blood heat is 98.

To fill a thermometer, heat the bulb and you will expel the air, then dip the other end into the fluid and it will immediately riſe and fill the bulb and part of the tube; and if there be any air bubbles, whirl it round about the upper end of the ſtem; and the centrifugal force of the fluid being greater than that of the air, the fluid will recede from

the center and drive out the air. Then heat the bulb and force the fluid to the top of the ſtem and hermetically ſeal it; and as the heat decreaſes, the fluid will fall and leave a vacuum above.

The preſſure of the atmoſphere againſt the outſide of the bulb, not being counteracted by any air within, affects its magnitude, diminiſhing it as the preſſure is increaſed. The variation however which this cauſes on the ſcale is never above one tenth of a degree.

81. If a piece of iron be heated and then left to be cooled by a current of air paſſing over it, in equal times, quantities of heat will be loſt in proportion to the whole quantity.

When the decrements of quantities vary as the quantities themſeves, theſe quantities muſt be in geometric progreſſion. Hence the heats retained, after equal intervals of time, are in geometric progreſſion. Sir I. NEWTON therefore heated a piece of iron red hot, and leaving it to cool, he laid upon it different metals and other fuſible bodies, and noted the times when by cooling they loſt their fluidity and began to coagulate; and laſtly, when the heat of the iron became equal to the external heat of the human body. Thus he extended the ſcale to all degrees of heat. See COTES's and PARKINSON's Hydroſtatics.

ON THE HYGROMETER.

82. Wood expands by moiſture and contracts by dryneſs; on the contrary, chord, catgut, &c. contract by moiſture and lengthen by dryneſs.

Hence by obſerving the expanſion and contraction of theſe ſubſtances, they will ſhow the different ſtates of the air in reſpect to moiſture. Various mechanical contrivances have been invented, to render ſenſible the ſmalleſt variations of the lengths of theſe ſubſtances. The twiſted beard of a wild oat, with a ſmall index fixed to it, moveable againſt a ſcale, makes a very good hygrometer; for the twiſting being affected by the variation of the moiſture of the air, it cauſes the index to move.

83. That ſubſtance is moſt proper for an hygrometer, whoſe expanſion or contraction varies moſt nearly in proportion to the quantity of moiſture imbibed.

Mr. DE LUC has made a great many experiments in order to find out thoſe ſubſtances whoſe expanſion increaſes moſt nearly in proportion to the quantity of moiſture imbibed. The reſult was, that whale-

bone

bone and box, cut a croſs the fibres, increaſed very nearly in proportion to the quantity of moiſture, and more nearly ſo than any other ſubſtances which he tried. This he found by taking a quantity of ſhavings of each ſubſtance, and weighing them at the time when he meaſured the increaſe of the length of a ſlip of each, cut as above deſcribed. He however preferred the whalebone, firſt, on account of its ſteadineſs, in always coming to the ſame point at extreme moiſture; ſecondly, on account of its greater expanſion, it increaſing in length above one eighth of itſelf from extreme dryneſs to extreme moiſture; laſtly, it is more eaſily made thin and narrow. He accordingly has conſtructed an hygrometer with whalebone, as the moſt accurate for the meaſure of the moiſture of the air. It is a little extraordinary that when he took threads of ſome ſubſtances in the direction of the fibres, they firſt increaſed as the quantity of moiſture increaſed, and afterwards upon a further increaſe of moiſture they decreaſed in length. See the *Phil. Tranſ.* for 1791.

ON THE PYROMETER.

84. All metallic bodies are expanded by heat.

Various inſtruments have been invented to render ſenſible very ſmall expanſions. If the rod to be expanded act very near to an axis of motion, by a proper combination of wheels to multiply velocity, the leaſt expanſion will be perceived and may be meaſured.

85. Rub a piece of metal with a cloth, and the warmth which it produces in the metal will ſenſibly increaſe its length.

86. If a lamp be put under a piece of metal, the metal will gradually increaſe in length as it grows hotter.

In this manner Mr. MUSCHENBROEK made experiments to determine the proportion of the expanſion of different metals, by applying a different number of lamps, and found the reſults as follows;

Lamps.	Iron.	Steel.	Copper.	Braſs.	Tin.	Lead.
1	80	85	89	110	153	155
2	117	123	115	220	*	274
3	142	168	193	275	*	*
4	211	270	270	361	*	*
5	230	310	310	377	*	*

Tin melted with two lamps and lead with three. With this kind of pyrometer Mr. FERGUSON found the expanſion of metals to be in the following proportion; iron and ſteel 3, copper $4\frac{1}{2}$, braſs 5, tin 6, lead 7. An iron rod 3 feet long is about one 70th of an inch longer in ſummer than in winter.

87. If

87. If a metal be put into water and the water be heated, the metal expands as the water increaſes in heat.

By this method Mr. SMEATON determined the expanſion of different metals, for by means of a mercurial thermometer immerſed in the water he could always aſcertain the degree of heat. He found that in equal intervals of time the expanſions were in geometric progreſſion. By this he was enabled to get the meaſure of the bar before it was applied to the inſtrument. This will be beſt underſtood by explaining an experiment. The time elapſed between applying the bar to the inſtrument and taking the firſt meaſure, was $\frac{1}{2}$ a minute; therefore the intervals between taking the ſucceeding meaſures were $\frac{1}{2}$ a minute alſo. The firſt meaſure was 208; the ſecond 214,5; the third 216,5; the fourth 217,5. The differences of theſe are 6, 5; 2; 1. Now theſe three numbers are nearly equal to 6, 3; 2, 25; 0, 8, which from a geometrical progreſſion whoſe common ratio is 2, 8. As therefore we may ſuppoſe the expanſion from the inſtant the bar was applied to the time of taking the firſt meaſure followed the ſame law, we can find the expanſion in the firſt $\frac{1}{2}$ minute (at the end of which the firſt meaſure was taken) by continuing back the progreſſion, or multiplying 6,3 by 2,8, which gives 17,7 for the lengthening the firſt $\frac{1}{2}$ minute; hence $208 - 17{,}7 = 190{,}3$ for the meaſure before the bar was applied. The following expanſions are ſelected from Mr. SMEATON's table, ſhowing how much a foot in length of each increaſes in decimals of an inch by an increaſe of heat correſponding to 180 degrees of FAHRENHEIT's thermometer, from freezing to boiling water. See Mr. SMEATON's account in the *Phil. Tranſ.* 1754.

White glaſs barometer tube	,01	Caſt braſs	,0225
Hard ſteel	,0147	Grain tin	,0298
Iron	,0151	Lead	,0344
Copper hammered	,0204	Zinc	,0353

Metals being thus ſubject to expanſion by heat, a pendulum made with a ſingle rod of metal will continually be ſubject to a variation in its length from the variation of the temperature of the air. To correct this Mr. HARRISON invented a pendulum, called a gridiron pendulum, compoſed of rods of iron and rods of braſs, ſo connected together, that the braſs expands upwards when the iron expands downwards; by this means the diſtance from the point of ſuſpenſion to the center of oſcillation is ſubject but to a very ſmall variation. Mr. GRAHAM invented the following method of preſerving the length of the pendulum the ſame in different temperatures. He took a glaſs, or metallick tube, and put in ſome mercury; now the heat, by expanding the glaſs or metal downwards, expanded the mercury upwards; by the adjuſtment therefore of a proper quantity of mercury, he could make theſe effects in altering the length of the pendulum nearly deſtroy each other. He found the errors of a clock of this ſort to be but about $\frac{1}{8}$ of the errors of the beſt clock of a common ſort.

OPTICS.

O P T I C S.

DEFINITIONS.

1. WHATEVER grants a paſſage to light is called a medium.

2. By rays of light is underſtood its leaſt parts, either ſucceſſive in the ſame lines, or cotemporary in ſeveral lines.

It is clear that light conſiſts of parts both ſucceſſive and cotemporary, becauſe in the ſame place you may ſtop that which comes one moment, and let paſs that which comes immediately after. The leaſt ſenſible part which may be ſtopped, or ſuffered to proceed, is called a ray of light.

3. *Refrangibility* is that diſpoſition of a ray of light to be refracted, or turned out of its courſe, when it paſſes out of one medium into another.

When a ray of light paſſes out of a rarer medium into a denſer, Sir I. NEWTON ſuppoſes that it is refracted by the ſuperior attraction of the denſer medium, and by that means drawn out of its courſe.

4. *Reflexibility* is that diſpoſition of a ray of light to be reflected, or turned back into the ſame medium from any other medium upon whoſe ſurface it may fall.

Sir I. NEWTON ſuppoſes that light is not reflected by impinging upon the ſolid parts of the body, but by ſome power of the body which is evenly diffuſed all over its ſurface, and by which it acts upon the ray and impels it back without immediate contact.

5. *Inflection* is that diſpoſition of a ray of light to be turned out of its courſe when it paſſes very near to the edges of bodies.

6. The angle of incidence is the angle which the line deſcribed by the incident ray makes with the perpendicular to the reflecting or refracting ſurface at the point of incidence.

7. The angle of reflection or refraction is the angle which the line deſcribed by the reflected or refracted ray makes with the perpendicular to the reflecting or refracting ſurface at the point of incidence.

8. Any parcel of rays diverging from a point, conſidered as ſeparate from the reſt, is called a *pencil* of rays.

9. A lens is a medium bounded by two ſpherical, or one plain and one ſpherical ſurface; and the line joining the centers, or which paſſes perpendicularly through each ſurface, is called the *axis*.

There are 6 lenſes, a double convex, a double concave, a plano-convex, a plano-concave, a concavo-convex and a meniſcus.

10. The focus of rays is that point from which they diverge, or to which they converge.

11. The focus of parallel rays is called the *principal* focus.

ON

ON THE GENERAL PROPERTIES OF LIGHT.

1. If the ſun's rays be admitted into a dark room perpendicularly through a circular aperture, they form a cone of bright light, decreaſing till it comes to the vertex, where the rays croſs, and it then increaſes; about this there is a kind of penumbra, or fainter light, which is terminated by lines drawn from the ſun to the oppoſite ſides of the aperture.

Hence the image of the ſun received within the room is a bright central light ſurrounded with a fainter light; and if $r =$ the radius of the aperture, $t =$ the tangent of $16'.2''$. the mean apparent ſemidiameter of the ſun; then the radius of the whole image at the diſtance x from the aperture $= r + tx$; alſo the radius of the bright central part $= r - tx$, or $tx - r$ according as you take it before or after the interſection. Hence when the radius of the aperture becomes evaneſcent the penumbra vaniſhes.

2. The image of an aperture of any figure will approach towards a circle as you receive it further from the aperture.

For the diameter $2r + 2tx$ of the image approaches to $2tx$ as its limit, by increaſing x; therefore however irregular the figure of the aperture may be, all the diameters of the image will approach to a ratio of equality, and conſequently the image will approach to a circle as its limit.

3. Lights which differ in colour have different degrees of refrangibility.

4. The ſun's light conſiſts of rays of different colours and differently refrangibile.

If the ſun's rays be admitted into a dark room through a ſmall hole in a window ſhutter, and be refracted through a priſm, the image is not round, but a long figure with parallel ſides and ſemicircular ends, the length of which is about five times its breadth; that end which has ſuffered the leaſt refraction is red, and that which has ſuffered the greateſt is violet. The whole image conſiſts of ſeven diſtinct colours, lying in the following order, red, orange, yellow, green, blue, indigo,

 violet;

violet; the red is the leaſt refrangible, and the other more in their order. Theſe are called primary colours, all other colours being only different combinations of theſe. Each colour forms a diſtinct image of the ſun, which images, in this experiment, running into each other, make a gradual change of colour in the image. But if a convex lens be placed before the priſm, each image will be diminiſhed, and by that means they will be ſeparated and each rendered diſtinct.

If two coloured images be formed with two priſms, and thrown one upon the other, then if that image be looked at through a priſm, the images will be again ſeparated.

5. The primary colours cannot be ſeparated into other colours by any refraction.

For if in the laſt experiment all the colours but one be ſtopped, for inſtance, the red, and that be again refracted by a priſm, it ſuffers no alteration in colour. By ſuffering the colours to paſs in ſucceſſion, from the red, each preſerves its colour, but the quantity of refraction keeps increaſing. The image of each colour is perfectly circular, which ſhows that the light of each colour is refracted regularly without any dilatation of the rays; it is therefore incompounded, or homogeneal.

6. If the breadth of each colour in the ſpectrum formed by the priſm be meaſured, it will appear that the breadth of the red, orange, yellow, green, blue, indigo, violet, are as the numbers 45, 27, 48, 60, 60, 40, 80, reſpectively.

If the circumference of a circle be divided into 45°, 27°, 48°, 60°, 60°, 40°, 80°, and the reſpective ſectors be painted red, orange, yellow, green, blue, indigo, violet, and the circle be turned ſwiftly, it will appear nearly white. For the ideas we have from the impreſſion of light remain for a ſhort time, and thus the colours excite the ſame ſenſation as if they all entered the eye collected together.

7. If the direct image of the ſun through a ſmall hole be received upon a ſkreen perpendicular to the rays, and the rays be then intercepted by a priſm and fall perpendicularly on the firſt ſide, if the diſtance from the place of the direct image to the neareſt edge of the red and fartheſt of the violet be meaſured, they will be the tangents of the angles of

of deviation, the radius of which is the diſtance from the point where the rays emerge to the place of the direct image.

The angle of incidence on the ſecond ſide of the priſm = the refracting angle of the priſm, to which add the deviations of the two extreme colours, and we get the two angles of refraction, the ſines of which will be to the ſine of incidence as 77 and 78 to 50. Hence if the difference between 77 and 78 be divided in the ratio of the breadth of each colour, it gives 77, $77\frac{1}{8}$, $77\frac{1}{5}$, $77\frac{1}{3}$, $77\frac{1}{2}$, $77\frac{2}{3}$, $77\frac{7}{9}$, 78 for the ſines of refraction, the common ſine of incidence being 50; that is, the ſine of incidence : the ſine of refraction of the red rays :: 50 : not leſs than 77 nor greater than $77\frac{1}{8}$, the boundary of the red; and the ſame for the reſt.

8. Candle light is of the ſame nature as the light from the ſun.

For rays from a candle may be ſeparated into all the different colours, and they lie in the ſame order as in the light from the ſun.

9. The ſun's light conſiſts of rays which differ in reflexibility, and thoſe rays which are moſt refrangible are moſt reflexible.

For after forming a coloured image, as before, with a priſm, by turning the priſm about its axis, until the rays within it, which in going out into the air were refracted at its baſe, become ſo oblique to the baſe as to begin to be totally reflected thereby, thoſe rays become firſt reflected, which before at equal incidences with the reſt had ſuffered the greateſt refraction.

10. According to Sir I. NEWTON, the colours of natural bodies ariſe from hence, that ſome reflect one ſort of rays and others another ſort more copiouſly than the reſt.

For every body looks moſt ſplendid in the light of its own colour, and therefore it reflects that the moſt copiouſly. Beſides, by reflection you cannot change the colour of any ſort of rays; and as bodies are ſeen by reflection, they muſt appear of the colour of thoſe rays which they reflect. This is the opinion of Sir I. NEWTON. But Mr. DELAVAL accounts for the colours of natural bodies in a manner different from this. See the *Mancheſter Memoirs*, Vol. II.

11. Thin tranſparent ſubſtances, as glaſs, water, air, &c. exhibit various colours according to their thickneſs.

For a very thin glaſs bubble, or a bubble of water, will appear to have concentric colours; the bubble blown with water, firſt made tenacious by diſſolving a little ſoap in it, continually grows thinner at the top by the ſubſiding of the water, the rings of colours dilating ſlowly, and overſpreading the whole bubble. A convex and concave lens of nearly the ſame curvature being preſſed cloſely together, exhibit rings of colours about the point where they touch. Between the colours there are dark rings, and when the glaſſes are very much compreſſed, the central ſpot is dark. Sir I. NEWTON, to whom we owe all theſe diſcoveries, found the thickneſs of the air between the glaſſes where the colours appeared to be as 1, 3, 5, 7, 9, &c. and the thickneſs where the dark rings appeared to be as 0, 2, 4, 6, 8, &c. The coloured rings muſt have appeared from the reflection of the light, and the dark rings from the tranſmiſſion of the light. The rays therefore were tranſmitted when the thickneſs of the air was 0, 2, 4, 6, 8, &c. and reflected at the thickneſſes 1, 3, 5, 7, 9, &c. Sir I. NEWTON therefore ſuppoſes, that every ray of light in its paſſage through any refracting ſurface is put into a certain conſtitution or ſtate, which in the progreſs of the ray returns at equal intervals, and diſpoſes the ray at every return to be eaſily tranſmitted through the next refracting ſurface, and between the returns to be eaſily reflected by it. Theſe he calls fits of eaſy tranſmiſſion and reflection.

12. If a beam of the ſun's light be let into a dark room, the ſhadow of an opaque body is larger than it ought to be, upon ſuppoſition that the rays of light proceded by it in ſtraight lines.

13. If the edges of two knives be placed parallel to each other at the diſtance of about the 400th part of an inch, and a ray of light fall upon them and ſome part paſs between their edges, the ſtream of light will part in the middle at the knives and leave a ſhadow between the two parts.

Hence it appears, that bodies act upon light at a diſtance; and in the caſe of the laſt propoſition it appears, that the body acted upon the light at the diſtance of the 800th part of an inch. In the former caſe the light was bent from the body, and in the latter towards it. This is called the *inflection* of a ray of light.

14. When

14. When a ray of light moves in a medium denser than its ambient medium, and comes to its surface and is reflected, the reflection will be stronger the rarer the ambient medium is; and the total reflection will take place at a greater angle of incidence the less the difference of the densities of the medium is.

For if the two mediums had the same density, it would be the same as the continuation of the same medium, in which case no reflection would take place; the reflection therefore will be the stronger by how much rarer the ambient medium is. Also, by prop. 31. the total reflection takes place after the sine of refraction becomes radius; now the sine of incidence : the sine of refraction, nearer to a ratio of equality the denser the ambient medium is, and therefore the sine of incidence must approach nearer to radius, as the limit, when the total reflection takes place, and consequently the angle of incidence becomes greater, the denser the ambient medium is, or the less the difference of the densities is. The refractive power is here supposed to vary as the density. Light therefore is not reflected by striking upon the solid parts of the surface of that body upon which it is incident, for if it were the denser ambient medium would cause the strongest reflection. It is therefore reflected by some power diffused over the surface.

15. The forces of bodies to reflect and refract light are nearly as their densities, except that unctuous and sulphureous bodies refract more than others of the same density.

For oil olive, spirit of turpentine and amber, which are sulphureous unctuous substances, have their refractive powers nearly as their densities; but their refractive powers are two or three times greater in respect to their densities than the refractive powers of bodies which are not sulphureous or unctuous.

ON THE FOCI OF, AND IMAGES BY, REFLECTED RAYS.

16. The angle of incidence is equal to the angle of reflection, and the plane passing through the incident and reflected rays is perpendicular to the surface.

17. Parallel

17. Parallel rays are reflected parallel.

In fact no pencil of rays can be accurately parallel; yet if the body from which the rays diverge be not nearer than a mile, all the rays in any pencil which we have ever any occaſion to conſider are, as to ſenſe, parallel.

18. Diverging rays reflected at a plane ſurface, after reflection will diverge from a point at the ſame diſtance on the other ſide, and in the ſame perpendicular.

19. If parallel rays fall very nearly perpendicularly on the concave ſide of a ſpherical reflector, after reflection they will all converge, very nearly, to the middle of that radius to which they are parallel: if they fall on the convex ſide, they will diverge from that point.

Hence the middle of the radius is the principal focus; it is alſo called the geometrical focus.

In optics, ſo far as regards practical purpoſes, we have occaſion only to inveſtigate the focus of rays falling very nearly perpendicularly upon the reflecting or refracting ſurfaces; for in practice the breadth of that ſurface is very ſmall in reſpect to the radius, and the rays fall nearly perpendicularly; the rules therefore here given, are ſufficiently accurate for the practical optician.

20. If the focus of diverging or converging rays, falling very nearly perpendicularly upon a ſpherical ſurface, and the center be joined, the diſtance of the principal focus from the focus of incident rays : the diſtance of the principal focus from the center or ſurface :: that diſtance : the diſtance of the principal focus from the focus of reflected rays upon the ſame line.

Cor. 1. Hence the diſtances of the foci of incident and reflected rays from the center are in the ſame ratio as their diſtances from the ſurface.

Cor. 2. Hence alſo if $d =$ the diſtance of the focus of incident rays from the ſurface, $r =$ the radius of the ſurface, the diſtance of the focus of reflected rays from the ſurface $= \frac{dr}{2d \backsim r}$.

21. The

21. The focus of reflected rays in the laſt prop. lies the ſame way from the principal focus as the focus of incident rays, but on different ſides of either the center or ſurface.

The two foci always move in oppoſite directions, and coincide at the center and ſurface.

22. If the focus of incident rays move, its velocity : the velocity of the reflected rays :: the ſquare of the diſtance of the principal focus from the focus of incident rays : the ſquare of half the radius.

23. If parallel rays be reflected at a ſpherical ſurface, the ſine of half whoſe arc $= s$, and radius $= r$; then when s is ſmall in reſpect to r, the longitudinal aberration from the geometrical focus $= \frac{s^2}{4r}$ nearly; and the lateral aberration $= \frac{s^3}{2r}$.

When the rays in a pencil diverge from a point, and either by reflection or refraction are brought all together again, they then form a luminous point correſponding to that from which they diverged. By this means a new viſible object is formed, called the image of the other; for the eye now receives the rays as coming from this latter point, and therefore it judges the former point to be in the place of the latter. And as this is true for every point of any object, every object may thus actually be formed anew, ſo far as regards our viſible ideas. And the rays diverging to the eye from the image thus formed, after the ſame manner as if they came directly from the object, excite an idea of that image, or of an object equal and ſimilar to it. Now if the pencils of rays which diverge from all the points of an object be again reſpectively collected at the ſame diſtances, they then form a new viſible object equal to that from whence they flowed; but if the points of this new object, called the image, correſponding to thoſe of the original object, be at a greater or leſs diſtance, they then form a new viſible object greater or leſs than the original one. Thus therefore we are able to form a new viſible object, very near to us, exactly ſimilar to an object at a great diſtance. I call this a viſible object, becauſe at the place where it is formed there are no correſponding tangible ideas, as in the object from whence the rays firſt flowed; but in reſpect to our viſible ideas, which we are here only conſidering, it is as much an object

object as the other. The eye therefore may be so situated in respect to this new object, that it may appear much greater than the original object, every object appearing greater the nearer it is to the eye. Now in respect to the brightness of this new visible object, we may consider, that when the eye looks directly at any object, it receives no more rays from any one point than what can enter the pupil; but when an image is formed by a lens, for instance, all the rays from any one point of the object which fall upon the lens are collected together and form a point of the image. Now if the diameter of the pupil of the eye = 0,1 in. and the diameter of the lens = 5 inches, their areas will be as 0,01 : 25, or as 1 : 2500; there are therefore, cæteris paribus, 2500 times as many rays collected together to form every point of the image by the lens as enter the eye and form the image, supposing all the rays to be refracted. Now although the rays diverge from every point of this image formed by the lens, and therefore where the eye is situated it may not receive them all, yet it being situated near to it, it will receive a very considerable part, and the more the nearer it is. Hence the number of rays which the eye receives from any point of this image may be greater than that which it receives directly from the object, and thus the image may be brighter than the object. These are the reasons why any distant object may be made to appear larger and brighter. And the common expression, that the object is brought nearer, is not incorrect; for the visible object is actually nearer; but it not being accompanied with any tangible ideas, we call it an image of the other; whereas it is a visible object formed by the same rays as the original visible object was. Looking therefore at the visible object thus formed, we get an idea of the original visible object seen under the same angle, and from thence, by association, we conclude what are the corresponding tangible ideas.

24. If any object be placed before a plane reflecting surface, the image will be equal to it, and situated at the same distance on the other side.

Cor. Hence if a man look at his own image in a plane reflector, it appears at the same distance on the other side, and is equal in magnitude to himself. Also if he view the whole of his image in a plane reflector, he appears to fill a space in the reflector equal to half his length and half his breadth, or one fourth of his area considered as a plane figure.

The best reflecting surfaces which we have are supposed to reflect not above one half the light which falls upon them; the rest enters and is lost.

25. If two plane speculums be placed parallel to each other, and any object be put between them, a number of images will appear in each speculum situated

ſituated one behind another in a perpendicular to the ſpeculums paſſing through the object.

If d be the diſtance of the ſpeculums, and x the diſtance of the object from one of them; then the diſtances of the images ſeen in that ſpeculum from that ſpeculum will be x, $2d-x$, $2d+x$, $4d-x$, $4d+x$, &c. and in the other ſpeculum the diſtances of its images from it will be $d-x$, $d+x$, $3d-x$, $3d+x$, &c. The further the images are from the ſpeculum the fainter they are.

26. If the ſpeculums be inclined to each other, a ſet of images will appear in the circumference of a circle whoſe radius is the diſtance of the object from the concourſe of the planes.

LEMMA.

If a be an arc of a circle of 1° to the radius 1, and m be to n as any other arc is to its radius; then $\frac{a}{1} : \frac{m}{n} :: 1^\circ : \frac{am^\circ}{n}$ the angle ſubtended by m.

27. If an object be ſpherical and concentric with a ſpherical reflector, the image will be ſpherical and concentric alſo with it.

Cor. 1. If the object be a line, the magnitudes of the object and image are as their diſtances from the center or ſurface, that is, as $d : \frac{dr}{2d \backsim r}$. If the object whoſe magnitude is m be ſituated between the principal focus and ſurface, and an eye be ſituated at a greater diſtance D from the ſurface, then, by the lemma, $\frac{a\,m^\circ}{D-d}$ and $\frac{a\,m\,r^\circ}{D \times \overline{r-2d}+dr}$ will be the apparent angles under which the object and image appear. Hence when $d = \frac{1}{2}r$, or the object be ſituated in the principal focus, the apparent magnitude of the image $= \frac{2am^\circ}{r}$, which is the ſame whereever the eye is placed. If the object be a ſpherical ſurface, we muſt take the duplicate ratio of $d : \frac{dr}{2d \backsim r}$ for the ratio of their magnitudes. The angles are here ſuppoſed to be ſmall.

Cor. 2. The object and image coincide, and conſequently become equal at the center and ſurface.

Strictly ſpeaking, when the object is at the ſurface it cannot be reflected to form an image, and at the center the object can be only a

 point;

point; the corollary therefore muſt be thus underſtood, that as the object approaches the center or ſurface, the image approaches it at the ſame time, and the diſtance between them keeps diminiſhing ſine limite.

28. The image is erect when it is on the ſame ſide of the center with the object, and inverted when on the contrary ſide.

When the object is beyond the center in reſpect to the ſurface, the image is between the center and principal focus, by prop. 21. and therefore it is inverted; but if the eye ſhould receive the rays reflected from the ſpeculum before the image is formed, the image would appear erect; then as the eye recedes from the ſpeculum, the image will grow confuſed, and when the eye gets to the place of the image after reflection, nothing diſtinct can then be ſeen, for the eye is then looking at an image cloſe to itſelf, and therefore there muſt be the ſame confuſion as when the eye looks at an object cloſe up to it; after that, as the eye recedes further back, the image will then begin to appear inverted, becauſe the eye will then look at an inverted image. And in general, although the reflector or lens may form an inverted image, if the rays enter the eye before the formation of the image, it appears erect; but when they enter the eye after the image is formed, it appears inverted.

29. The image of a ſtraight line by reflection at a ſpherical ſurface is a conic ſection.

If the object be placed at the diſtance of half the radius of the reflector from the center, the image will be a parabola; if further from the center, an ellipſe; if nearer to the center, an hyperbola.

ON THE FOCI OF, AND IMAGES BY, REFRACTED RAYS.

30. When a ray of light paſſes out of one medium into another, the ſine of incidence is to the ſine of refraction in a given ratio.

We are here to underſtand rays of the ſame colour. The ſines of incidence and refraction of the moſt and leaſt refrangible rays out of glaſs into air are as 50 : 77 and 78; hence for the mean rays the ratio is 20 : 31, or nearly as 2 : 3, which is, in common, taken for the ratio of all rays. Out of rain water into air, theſe ratios are as 81 : 108 and 109; for the leaſt refrangible rays therefore the ratio is 3 : 4, which, in common, is taken for that of all the rays.

31. Light

31. Light cannot paſs out of a denſer medium into a rarer, when the ſine of incidence has a *greater* proportion to radius than it has to the ſine of refraction.

The limit is when the proportion is the *ſame*. Hence a ray cannot paſs out of water into air at a greater angle of incidence than 48°.36′. the ſine of which $= \frac{3}{4}$ of radius. Out of glaſs into air the angle muſt not exceed 40°.11′. When the angle however is within the limit for the light to be refracted, ſome of the rays are reflected.

32. As the angle of incidence increaſes, the angle of deviation will increaſe.

33. Parallel rays of the ſame colour falling upon a plane refracting ſurface, will continue parallel after refraction.

34. Parallel rays of the ſame colour paſſing through a medium bounded by plane parallel ſurfaces, will continue parallel.

35. If parallel rays paſs through a priſm whoſe refracting angle is ſmall, and the angle of incidence be alſo very ſmall, and I and R be the angles of incidence and refraction, G the refracting angle, then the angle of deviation after the rays have paſſed through the priſm $= \frac{G \times \overline{I-R}}{R}$.

Cor. Hence the deviation is in proportion to the refracting angle.

36. If diverging or converging rays fall very nearly perpendicularly upon a plane refracting ſurface, the diſtance of the focus of incident rays from the ſurface : the diſtance of the focus of refracted rays :: the ſine of refraction : the ſine of incidence, or as $R : I$.

M 2 We

We ſhall uſe $I : R$ to expreſs the ratio of the ſine of incidence : the ſine of refraction.

37. If diverging rays paſs nearly perpendicularly through a medium bounded by two plane parallel ſurfaces, the diſtance between the foci of incident and emergent rays : the diſtance of the ſurfaces :: $I : I - R$, I and R being the ratio of the ſines of incidence and refraction at the firſt ſurface.

38. If parallel rays fall very nearly perpendicularly on a ſpherical refracting ſurface, the diſtance of the focus of refracted rays from the ſurface : the radius of the ſurface :: $I : I - R$ or $R - I$.

This focus is called the principal focus.

39. If diverging or converging rays fall very nearly perpendicularly upon a ſpherical refracting ſurface, the diſtance of the focus of incident rays from the principal focus : its diſtance from the center :: its diſtance from the ſurface : its diſtance from the focus of refracted rays.

Cor. 1. Hence the two foci coincide at the center and ſurface.

Cor. 2. The four diſtances in the propoſition lie all the ſame way from the focus of incident rays, or two on each ſide.

A concave ſurface of a denſer medium and a convex of a rarer give a divergency to rays; and a convex ſurface of denſer and concave of a rarer medium give a convergency.

40. If parallel rays fall very nearly perpendicularly upon a ſphere, and that diameter to which they are parallel be produced, if neceſſary; the focus after the rays have paſſed through the ſphere will biſect the diſtance between the focus at the firſt ſurface and the extremity of the above diameter which is on the contrary ſide to the incident rays.

If

If $r =$ the radius of the ſphere, the diſtance of the principal focus from the center $= \frac{rI}{2I-2R}$. If the ſphere be water, the focus lies at the diſtance of a radius beyond its ſurface; if glaſs, that diſtance is half the radius.

41. If the line joining the centers of the ſurfaces of a lens be divided in the ratio of the reſpective radii, all the rays paſſing through that point will have their incident and emergent parts parallel.

That point is called the *center* of the lens.

Hence if the thickneſs of the lens be inconſiderable, the paſſage of every ray which paſſes through the center may, for all practical purpoſes, be conſidered as a ſtraight line.

42. If the ſine of incidence out of air into a double convex or concave lens : the ſine of refraction :: $I : R$, and the radii of the two ſurfaces be m and n, the diſtance D of the principal focus from the center of the lens $= \frac{mn}{m+n} \times \frac{R}{I-R}$, the thickneſs of the lens being inconſiderable, and the rays falling nearly perpendicularly.

Cor. 1. If the lens be glaſs, and I be taken to R :: 3 : 2, then $D = \frac{2mn}{m+n}$; and if $m = n$, $D = m$.

Cor. 2. If one radius n become infinite, or the lens become plano-convex or concave, $D = m \times \frac{R}{I-R}$. Hence for ſuch a glaſs lens, $D = 2m$.

Cor. 3. The focal length is the ſame on which ever of the ſides the rays fall, the radii being ſimilarly involved. Alſo, all lenſes of the ſame focal length muſt have the ſame effect.

43. If the lens be a meniſcus, or concavo-convex, $D = \frac{mn}{m-n} \times \frac{R}{I-R}$.

Theſe

These rules, though not mathematically true, are sufficiently accurate for all practical purposes.

Parallel rays on a convex lens, plano-convex lens and meniscus, converge to the principal focus; but on a concave lens, plano-concave and concavo-convex lens, they diverge from the principal focus. As therefore the rays would return back in the same lines, rays diverging from the principal focus in the former case, and converging to it in the latter, become parallel after passing through the lens.

To find the principal focal length of a convex lens, hold it parallel to a skreen which is perpendicular to the sun's rays, and remove it backwards and forwards till you find the bright spot the least you can make it, and the distance of the lens from the skreen is its focal length. Or remove it till the image be equal to the lens, and the distance is equal to twice the focal length. If the lens be concave, remove it from the skreen till the bright annulus surrounding a darker central circle be equal in diameter to twice the diameter of the lens, and the distance of the lens from the skreen is the focal length. The circle in the center is darker than that part without, because it receives only those rays which pass through the lens, whereas the annulus beyond receives those rays which pass through the lens, and the direct rays of the sun also. The part beyond the annulus receives the direct rays only, and therefore is darker than the annulus, but it is brighter than the central circle, because the direct rays after refraction have their density diminished by being rendered diverging. Hence therefore the quantity of light in the annulus is equal to the sum of the quantities on each side.

44. If diverging or converging rays fall nearly perpendicularly upon a lens, the distance d of the focus of incident rays from the principal focus : its distance from the center :: that distance : the distance D of the focus of incident and refracted rays.

Hence, and from prop. 42. $d \backsim \frac{mn}{m+n} \times \frac{R}{I-R} : d :: d : D =$ $\frac{\overline{m+n} \times \overline{I-R} \times d^2}{\overline{m+n} \times \overline{I-R} \times d \backsim mnR}$, for a convex lens; for a concave $D =$ $\frac{\overline{m+n} \times \overline{I-R} \times d^2}{\overline{m+n} \times \overline{I-R} \times d + mnR}$. Hence the distance of the focus of refracted rays from the lens $= \frac{mnRd}{\overline{m+n} \times \overline{I-R} \times d \mp mnR}$. Hence the distance of the focus of incident rays from the lens : the distance of the focus of refracted rays from the lens :: $\overline{m+n} \times \overline{I-R} \times d \mp mnR : mnR$. For a glass lens, this ratio becomes $\overline{m+n} \times d \overset{\backsim}{+} 2mn : 2mn$.

The

The focus of refracted rays lies the ſame way from the focus of incident rays as the principal focus does.

By moving the focus of incident rays the focus of refracted rays moves in the ſame direction; and they meet at the lens.

The proportion in the propoſition holds for rays falling on a ſphere, only aſſuming its principal focal length from the center, inſtead of the principal focal length of the lens.

45. If a medium be bounded by an ellipſe or hyperbole revolving about its major axis, and the major axis : the diſtance of the foci :: $I : R$, all rays parallel to the major axis, entering into the former or coming out of the latter, will be refracted to the other focus.

46. Let the ſine of incidence be to the ſine of refraction of the leaſt and moſt refrangible rays, as p to m and n reſpectively, then if parallel rays fall on a plano-convex lens, the diameter of the aperture : the diameter of the circle of aberration in the focus of the lens :: $n - m : n + m - 2p$.

Cor. 1. Hence if the lens be glaſs, p, m and n are 50,77 and 78 reſpectively; hence the diameter of the circle of aberration $= \frac{1}{55}$th part of the diameter of the aperture.

Cor. 2. Hence alſo the angle of aberration varies as the diameter of the aperture directly and the focal length inverſely.

47. If parallel homogeneal rays fall upon the plane ſide of a plano-convex lens, and $n : m$ :: the ſine of incidence out of glaſs into air : the ſine of refraction, alſo let $r =$ the radius of the aperture of the lens, and $d =$ the diſtance of the principal focus from the ſurface of thoſe rays which fall very near the center; then the longitudinal aberration $= \frac{m^2}{\overline{m-n}^2} \times \frac{r^2}{2d}$, and the lateral aberration $= \frac{m^2}{\overline{m-n}^2} \times \frac{r^3}{2d^2}$.

48. The

48. The diameter of the circle of aberration is equal to half the lateral aberration in the laſt propoſition.

If the lens be glaſs, and we take $n : m :: 20 : 31$, alſo if $r = 2$ inches, and the radius of the ſurface $= 600$ inches, the diameter of the circle of aberration $= \frac{31^2 \times 8}{20^2 \times 4 \times 600^2} = \frac{961}{72000000}$ of an inch. This is the aberration from the ſpherical form of the glaſs. Now the aberration from the different refrangibility of the rays $= \frac{4}{55}$ by prop. 46. Hence the former aberration : the latter :: 1 : 5449. The aberration therefore from the form of the glaſs is ſo ſmall that it may be neglected. Before Sir I. NEWTON, the imperfection of refracting teleſcopes was ſuppoſed to ariſe from the ſpherical figures of the glaſſes; but he has thus ſhown that it ariſes principally from the different refrangibility of light. He propoſed therefore to form the image of the object by reflection, which would not be ſubject to a like imperfection, and for this purpoſe he conſtructed a reflecting teleſcope.

As the aberration of rays from the different refrangibility of light is ſo great, it might be expected that an image could not be formed ſo free from colour as we find it; but the rays are not ſcattered uniformly over the circle of aberration, but are much more denſe towards the middle, the denſity varying as the diſtance from the circumference directly and diſtance from the center inverſely, nearly. On account therefore of their quick increaſe of rarity as you recede from the center, only thoſe rays which are near the center are ſtrong enough to be viſible.

49. An object in the water looked at in a direction perpendicular to the ſurface appears elevated $\frac{1}{4}$ of its depth in the water.

As the eye recedes from that perpendicular the object appears to riſe, and would appear at the ſurface if the eye were removed to an infinite diſtance. If a river be 6 feet deep it appears to be only $4\frac{1}{2}$ ft.; a perſon not appriſed of this might venture into water at the hazard of his life.

50. The image of a ſtraight line by refraction at a plane ſurface, is a ſtraight line, but not equal to the object, or ſimilarly ſituated, unleſs the object is parallel to the ſurface.

All plane objects will be ſimilar to their images when they are parallel to the ſurface; otherwiſe not. Hence alſo a ſtraight rod put into water appears bent at the ſurface, the image of the part within lying above the object.

51. An object ſeen through a medium bounded by two plane parallel ſurfaces whoſe diſtance is d, appears nearer by $d \times \frac{I-R}{I}$ by prop. 37.

For the image of an object is formed by the images of every point of the object. If the medium be glaſs it appears nearer by $\frac{1}{3}d$; therefore an object ſeen through common glaſs does not appear to be ſenſibly altered.

52. An object ſeen through a very ſmall hole appears inverted, becauſe the rays from the extreme parts of the object muſt have croſſed at the hole.

The rays of light are probably inflected as they paſs by the edge of the hole, for it renders diſtant objects very diſtinct to a ſhort ſighted perſon, and has therefore the property of a concave lens. This effect however may perhaps partly ariſe from the ſmall number of rays admitted to the eye, by which the confuſion to a ſhort ſighted perſon may in ſome meaſure be taken off. But from my own experience of the appearance of objects thus ſeen, I can have no doubt but that the former effect is produced.

53. The image of a ſpherical object concentric with a lens, will be alſo ſpherical and concentric with it.

54. The image will be erect or inverted, according as it is on the ſame or different ſides of the center in reſpect to the object.

In a convex lens the image will be erect when it is further from the lens than the principal focal length, and inverted when nearer. In a concave lens the image is always erect.

When the image is inverted, if the eye receive the rays before they form the image, the object appears erect; then as the eye recedes from the lens, a confuſion will come on till the eye gets to the image, where nothing will appear; afterwards the object appears inverted.

55. If r be the principal focal length, d the diſtance of the object from the lens, then the diſ-

tance of the image from the lens $= \frac{dr}{d \underset{+}{\infty} r}$, where the lower or upper sign takes place according as the lens is concave or convex.

In a concave lens the image is always nearer than the object; in a convex lens it may be nearer or further off.

56. If the object be an arc of a circle, the magnitude of the object : the magnitude of the image :: $d \underset{+}{\infty} r : r$; but if it be a spherical surface, their magnitudes are as $\overline{d \underset{+}{\infty} r}^2 : r^2$.

57. The apparent diameters of an object to the naked eye, and seen through a lens, are as their magnitudes divided by their distances from the eye.

An object always appears diminished when seen through a concave lens, and magnified in a convex lens when nearer than the principal focus, unless the lens is close to the object or eye, in which cases the apparent magnitude is not altered. If a linear object whose magnitude is v be placed beyond the principal focus of a convex lens, and the eye be on the other side at the distance m from the lens, the apparent magnitude to the naked eye, and the apparent magnitude in the lens will, by the lemma, be $\frac{av^\circ}{d+m}$ and $\frac{avr^\circ}{dr - \overline{d-r} \times m}$. Now if $d = r$, or the object be situated in the principal focus, these become $\frac{av^\circ}{m+r}$ and $\frac{av^\circ}{r}$. Which shows that as the eye recedes from the lens, the apparent magnitude of the object to the naked eye diminishes, and the apparent magnitude of the image will remain the same. If $m = r$, the magnitudes become $\frac{av^\circ}{r+d}$ and $\frac{av^\circ}{r}$; hence the apparent magnitude of the image is the same at all distances of the object from the lens. In all these cases the angles are supposed to be very small.

In vision by images, as Mr. HARRIS in his excellent Treatise of Optics observes, we are generally deprived of many circumstances by which we usually judge of distance, which makes it very difficult in most cases to judge of the place of an image, if it be further off than 2 or

2 or 3 yards. These difficulties are again increased by some peculiarities belonging to images, which we are not accustomed to observe, and for which therefore we are at a loss how to make proper allowances.

In respect to the idea of apparent magnitude, as the same author observes, it is difficult to ascertain precisely, how far, or in what proportion, apparent distance affects it. The visual angle may be the same and yet the apparent magnitude very different: a pane of glass, for instance, does not appear so big as the front of an house seen through it; nor does a child 2 feet high appear so big as a man 6 feet high at three times the distance, although in both cases the visual angle is the same. If we suppose an object to be at a greater distance, and subtend the same angle as one at a less distance, we have an idea of a greater apparent magnitude. Mr. Harris therefore supposes, that *apparent magnitudes are either exactly, or very nearly, in the compound ratio of the visual angles and apparent distances*. Hence when objects are so near that the apparent distances are judged to be the same as the true, the apparent magnitude is not altered by altering its distance: thus a man appears as big at the distance of 6 yards as he does at 1 yd. And it seems to be necessary, that we should have some certain means of judging of near distances, otherwise we might be frequently in great danger without perceiving it. But when, by increasing the distance of an object, or its image, the apparent distance does not increase so fast as its true distance, the apparent magnitude diminishes. For if $m =$ the magnitude of the object, or its image, $d =$ the real distance from the eye, $D =$ the estimated distance; then the apparent magnitude $= \frac{m}{d} \times D$; hence, as long as $D = d$, the apparent magnitude is the same at all distances; but when D increases slower than d, which is the case when they become of considerable magnitude, then the apparent magnitude diminishes. Hence, as we are led to judge of the situation of an object, and more particularly so of an image, to be so very different from their true place, it happens that our ideas of apparent magnitude differ so much from the visual angles. This extends to all images both by reflection and refraction. The reader will find a great deal of satisfaction upon the subject in Mr. Harris's Optics.

58. If in the place of a real image, that is, an image formed by rays converging, a white paper be placed, the image will be formed upon the paper.

For the rays are then reflected from every point of the image in all directions, in like mannner as if it were a real object But when the paper is removed, the rays proceed only straight forward, and therefore the image can only be seen by an eye placed directly behind it.

59. The image of a circular object concentric with a single spherical refracting surface, will be also circular and concentric with it.

 60. If

60. If the object be linear, its magnitude and the magnitude of the image will be as their distances from the center.

61. The image will be erect or inverted, according as it is on the same or different sides of the center in respect to the object.

62. The image of a straight line by refraction through a lens, is a conic section.

If the object be situated in the principal focus, the image is a parabola; if nearer to the lens, it is an hyperbola; if further from the lens, an ellipse.

63. If a ray of light pass out of air into any medium at an angle of incidence whose sine $= s$, and the sines of refraction of the least and greatest refrangible rays be m and n; then the arc, to radius unity, measuring the whole dispersion of the rays $= \frac{n-m}{c}$, c being the cosine of refraction of the mean refrangible rays.

For the same medium $m : s :: 1 : v$ a constant ratio, and $n : s : 1 : w$; hence the dispersion $= \frac{s \times \overline{v-w}}{cvw}$. Let rays pass out of air into flint glass, and $v = 1{,}565$, $w = 1{,}595$; let rays also pass out of air into common glass, at the same angle of incidence, then $v = 1{,}56$, $w = 1{,}54$; now as we may consider c to be the same in both cases, we have the dissipating powers of these two mediums at the same angle of incidence as $\frac{0{,}03}{1{,}565 \times 1{,}595} :: \frac{0{,}02}{1{,}54 \times 1{,}56} :: 3 : 2$ very nearly.

When the rays fall very nearly perpendicularly, we may consider s and c as constant, and the dissipating powers will in that case, be always as $\frac{v-w}{vw}$.

64. If a ray of light pass through a prism and both refractions be the same way, the whole dispersion

persion of the rays will be the sum of the dispersions at going in and coming out of the prism, each of which may be computed by the rule in the last proposition, having given the angle of incidence upon the first surface and the refracting angle of the prism.

If s = the sine of incidence on the first surface, c = cosine of refraction, S = the sine of incidence on the second surface, C = the cosine of refraction, v and w as in the last proposition; then the whole dispersion $= \frac{s \times \overline{v-w}}{c \times vw} + \frac{S \times \overline{v-w}}{C \times vw} = \frac{sC + Sc}{Cc} \times \frac{v-w}{vw}$. If the refracted ray within the prism be parallel to the base, s and C are the sines and cosines of the same angle, and so also are S and c.

65. Two prisms made of different kinds of glass may have their refracting angles so adjusted, that when the refracting angle of one is applied to the base of the other, a ray of light passing through them shall have its incident and emergent parts parallel, and the emergent part shall be coloured.

66. Two prisms may be made and applied as before, and the emergent ray shall be free from colour, but not parallel to the incident ray.

Sir I. NEWTON, after having determined the proportion of the sine of incidence to the sines of refraction of different coloured rays, as given by his glass prisms, procedes to discover their proportions in different refracting mediums. He placed a prism of glass in a prismatic vessel of water, and refracting the light through these mediums, he found that light, as often as by contrary refractions it was so corrected that it emerged in lines parallel to those in which it was incident, continued to be white; but if the emergent rays were inclined to the incident, the light became coloured. The conclusion from this experiment was, that the divergency of the different coloured rays was constantly in a given ratio to the mean refraction in all mediums. But in the year 1757, Mr. DOLLOND tried the same experiment and found the result to be very different; for when the light was refracted in contrary directions through the glass and water prisms, if the emergent rays were parallel to the incident rays, they were found to be considerably coloured; from whence it followed that the dispersion of the

the rays of different colours was not in a constant ratio to the mean refraction in water as in glass, because there was a dispersion without any mean refraction. And further experiments proved that there was also a very considerable difference of the same kind to be found in different sorts of glass. This discovery of Mr. DOLLOND was so extraordinary, and so contrary to the best established principles, that Mr. EULER did not at first believe it. At length however Mr. ZEIHER, at *Petersburg*, made experiments of a similar kind, and convinced Mr. EULER that it was true. He showed that it is the lead, which is used in some compositions of glass, which produces that very extraordinary property of augmenting the dispersion of the extreme rays, without sensibly changing the refraction of the mean. Mr. EULER, in a paper read at the Academy of Sciences at Berlin in 1764, was candid enough to confess that he did not at first credit the account, and thereby gave to Mr. DOLLOND the honour of the discovery. Notwithstanding this declaration of Mr. EULER, M. DE LA LANDE in his Astronomy published in 1764, and FUSS in his Eulogy on EULER, both ascribe the invention to EULER. But it still remains to be explained from whence arose this difference between Sir I. NEWTON's and Mr. DOLLOND's experiments. This Mr. P. DOLLOND has done in a pamphlet, entitled, *Some account of the discovery by the late* Mr. JOHN DOLLOND, F.R.S. *which led to the great improvement of refracting telescopes.* In NEWTON's time the English were not the most famous for making telescopes, and a great many were imported from *Italy*, and particularly from *Venice*. The glass made at *Venice* was nearly of the same refractive quality as our crown glass, but of better colour. It is probable that NEWTON's prisms were made of that glass, because he mentions the specific gravity of common glass to be to water as 2,58 : 1, which nearly answers to that of Venetian glass. Now Mr. DOLLOND had a piece of that glass by him, of which he made a prism, and trying the experiment with it, he found it answered very nearly to what NEWTON relates, the difference being only such as may be supposed to arise from the same kind of glass made at different times. Hence it appears, that NEWTON was accurate in his experiment, and had he used prisms of different glass, he would have made the discovery which led to the perfecting of refracting telescopes.

Mr. DOLLOND having discovered that different kinds of glass had different dispersive powers, examined all the different kinds, and found that the difference of dispersions was greatest in the *crown* and *white flint* glass. He therefore ground a wedge of flint glass at an angle of about 25°, and several others of crown glass, till he found one with the same dispersive power as the flint glass, and applying these together so as to refract in contrary directions, he found that the emergent rays were free from colour but not parallel to the incident rays. They were free from colour because the dispersive powers were equal and contrary, but not parallel to the incident rays, because the mean refractive power of each prism was different. In like manner he found that he could apply a wedge of crown glass to the flint which should have the same mean refractive but a different dispersive power, by which means the emergent rays would be parallel to the incident, but

would

would be coloured. They would be parallel becaufe the mean refractive powers were equal and contrary, and coloured becaufe the difperfive powers were unequal. Thus he could produce refraction without difperfion, and difperfion without refraction.

Mr. DOLLOND next confidered, that as a ray might be refracted free from colour through a wedge, it might alfo through a lens. When an image of an object is formed by a convex lens, it appears coloured, owing to the difperfion of the rays by refraction; as therefore rays can be refracted without difperfion by prifms, he conceived that it might alfo be done by a combination of lenfes, And in this he fucceeded, by confidering, that in order to make two fpherical glaffes that fhould refract the light in contrary directions, as in the two wedges, one muft be concave and the other convex; and as the rays are to converge to a real focus, the excefs of refraction muft be in the convex lens, becaufe that makes rays converge and the concave makes them diverge. Alfo, as the convex lens is to refract moft it muft be made of crown glafs, as appeared from the experiments with the wedges, and the concave of white flint glafs. Farther, as the angle of difperfion varies inverfely as the focal length, very nearly, from the principles of optics, and the angle of difperfional fo varies as the difperfing powers, therefore if the focal lengths be taken inverfely as the difperfing powers, found from the two wedges, the angles of difperfion will be equal, and being in contrary directions they will correct each other and the different refrangibility of light will be removed. Upon this principal Mr. DOLLOND, was enabled to make a combined lens to form an image free from colour, and therefore brought to perfection the refracting telefcope, making it reprefent objects with great diftinctnefs, and in their true colours. Inftead of forming the object glafs with one convex lens of crown and one of flint glafs, two convex lenfes of crown are ufed and the concave one of flint put between them. This conftruction of the object glafs tends alfo to correct the error arifing from the fpherical form of the lens; for as the rays at the edge of the convex lens tend to a focus nearer to the lens than thofe at the middle, the concave lens, which makes the rays at the edge diverge more than thofe at the middle, will counteract the above effect, and bring the rays at all diftances from the center of the lens to a focus more nearly together; and by a proper adjuftment of the foci the diffufion of rays at the focus may be rendered inconfiderable. Telefcopes thus conftructed are called *Achromatic*.

ON THE CONSTRUCTION OF OPTICAL INSTRUMENTS.

67. A fingle microfcope is formed by one convex lens, having the object in the principal focus, and the linear magnifying power is equal to the leaft diftance

diſtance at which the naked eye can ſee diſtinctly divided by the focal length of the lens.

In this, and alſo in the next proportion, we compare the magnitude ſeen through the glaſs with that ſeen without it at the leaſt diſtance of diſtinct viſion, which is uſually about 6 or 7 inches.

68. A compound microſcope is formed with two convex lenſes; the object is placed a little beyond the principal focus of one, by which means a large inverted image is formed, and the eye glaſs is placed at the diſtance of its focal length from the image; by this combination, the linear magnifying power is equal to the leaſt diſtance of diſtinct viſion multiplied by the diſtance of the image from the object glaſs, divided by the diſtance of the object from the object glaſs multiplied by the focal length of the eye glaſs.

The object appears inverted becauſe the eye looks at an inverted image. The brightneſs of the object is as the magnitude of the object glaſs; and the field of view as the magnitude of the eye glaſs, all the other circumſtances being the ſame.

69. A ſolar microſcope is formed by two convex lenſes, one of which is to receive the rays of the ſun and throw them upon the object to illuminate it; and the object being ſituated a little beyond the principal focus of the other lens, a large inverted image is formed and received upon a ſkreen, and magnified, in linear dimenſions, as the diſtance of the image from the lens divided by the diſtance of the object from the lens.

70. A magic lanthorn is formed by placing an object a little beyond the principal focus of a convex lens, and receiving the image upon a ſkreen; it therefore magnifies as the ſolar microſcope.

The

The object is illuminated by a lamp or candle, and the rays are generally received on the plane side of a glass segment of a sphere and refracted to the object, in order to illuminate it more strongly.

71. A camera obscura is a box with a convex lens put into a moveable tube, by which the images of distant objects are formed upon a plane in the box proper to receive them.

This should be used when the sun shines, otherwise the image will be faint.

72. The astronomical telescope consists of a double convex lens object glass and eye glass, and the magnifying power is equal to the focal length of the object glass divided by that of the eye glass.

The distance of the two glasses = the sum of their focal lengths; and an inverted image being formed by the object glass is looked at by the eye glass. This is only used for astronomical observations, because the objects appear inverted, which, for such purposes, is of no consequence.

The brightness of the object is as the magnitude of the object glass; and the visible area as the magnitude of the eye glass, the other circumstances remaining the same. By proportioning the focal distances of these glasses, you may magnify as much as you please; but unless you can increase the quantity of light in proportion, the object will become indistinct for want of light. Now you can increase the quantity of light only by increasing the magnitude of the object glass, which you cannot do, if single, to a great degree without the object appearing coloured from the different refrangibility of the rays of light. Here then is the great use of the achromatic object glass, which admits of being very large without separating the colours, by which you can form a very perfect strong image.

73. The terrestrial telescope is composed of an object glass of a double convex lens, and three eye glasses, generally of the same focal length, and magnifies, in that case, as the astronomical telescope.

The object glass first forms an inverted image, and then the two eye glasses next to it receive the rays and invert the image again, and the third eye glass is to look at this erect image. Hence the object appears erect, and therefore is fit for terrestrial observations. In the best telescopes the object glass is achromatic, and forms a colourless image; but it might here be expected that the second image formed by the eye glasses, they not being achromatic, would be coloured, which is not the case; the reason is, that the aperture of the eye glasses is but small,

and the dispersion of the rays in the focus is as the aperture, and therefore here is but small, and that may be removed by a proper adjustment of the eye glasses, and increasing their number. See Mr. RAMSDEN's Paper on this subject in the *Phil. Trans.* 1783.

74. GALILEO's telescope is formed by a convex lens object glass, and concave eye glass, whose distance is the difference of their focal lengths, and the magnifying power is equal to the focal length of the former divided by that of the latter.

The eye glass intercepts the rays before the image is formed by the object glass, and therefore the object appears erect. The brightness is as the magnitude of the object glass; and the visible area is as the magnitude of the pupil of the eye, and will be greater also the nearer the eye is to the glass, all other circumstances being the same.

75. Sir ISAAC NEWTON's reflecting telescope is formed by placing a concave reflector to receive the rays and form the image; but those rays are received after reflection, before the image is formed, upon a plane reflector making an angle of 45° with the axis of the reflector, by which means the image is thrown out of the axis and lies parallel to it, where it is looked at by an eye glass. The magnifying power of this telescope is the focal length of the reflector divided by that of the eye glass.

The brightness is as the magnitude of the reflector; and the field of view as the magnitude of the eye glass, all other circumstances being the same.

This telescope was constructed by Sir I. NEWTON in order to form an image free from colour, which at that time could not be done by refraction. Since therefore the invention of the achromatic telescopes, these have been of less use; for the image, although free from colours, is not so sharp and distinct as by refraction. Instead of a plane speculum to reflect the light, a right angled prism is sometimes used, so that the rays enter and go out of the two sides including the right angle perpendicularly, and are reflected at the third side, which reflection is stronger than from a plane speculum. The third side is not quicksilvered over, for without quicksilver it will reflect all the light incident upon it from the speculum.

Since

Since the time of Sir I. NEWTON, reflecting telescopes have been differently constructed. Dr. GREGORY formed the image by a concave reflector, and then at a little distance beyond the image from that reflector he placed another concave reflector which formed a second image, which is viewed through a hole in the center of the first reflector by a convex eye glass. Mr. CASSEGRAIN afterwards made a small alteration in the construction, by placing a small convex reflector to receive the rays before the image was formed by the first reflector, by which he formed an image which was viewed as before. This construction has some advantage over the former, for as one reflector is convex and the other concave, the error arising from the spherical form of the first reflector is partly corrected by the second; whereas in the other form it would be increased.

76. When a short sighted person uses a telescope, he must push the eye glass nearer to the object glass; the contray for a long sighted person.

77. The density of light varies inversely as the square of the distance from the point from which it diverges.

Hence the quantity of light received by a telescope from an object varies inversely as the square of the distance of the object from the telescope, and consequently the brightness of the object seen through it must vary in the same ratio, every thing else being the same. By density we do not here mean the number of particles in a given space of three dimensions, but the number on a given plane.

ON THE RAINBOW.

78. If a ray of light enter a sphere of a denser medium, and after n reflections within it emerge; then if $a=$ the angle of incidence, $b=$ the angle of refraction at the entrance, the angle under the incident and emergent rays $=2a-\overline{2n+2}\times b$.

When the number of reflections is odd, it is the angle between the rays; when the number is even, it is the angle on the other side.

79. When the angle under the incident and emergent rays is a maximum or minimum, the tangent of the angle of incidence : the tangent of refraction :: $n+1$: 1.

As the medium, in the following propofitions, is fuppofed to be water, the fine of incidence : the fine of refraction for the leaft refrangible rays as 77 : 50, and for the greateft as 78 : 50.

80. The ratio of the fines and tangents of incidence and refraction being given as in the two laft propofitions, the angles themfelves may be found.

81. The rainbow is formed by the refraction and reflection of the fun's rays on falling drops of rain.

82. Rays are faid to be efficacious when a fufficient number of any one colour comes to the eye to excite the idea of that colour.

83. Rays are moft efficacious when thofe of the fame colour emerge parallel.

For then they keep together and all enter the eye. When the rays emerge parallel, they will, when produced to meet the incident rays, make equal angles with them.

84. If they make equal angles with each other, that angle muft be either a maximum or minimum.

For when a quantity is either a maximum or minimum, it is at that time neither in an increafing or decreafing ftate.

85. Hence the angle of incidence found in prop. 80. is the proper angle to render the rays efficacious.

86. As the rays of different colours have different degrees of refrangibility, the angle of incidence, to render the rays efficacious, muft be different for the different colours.

87. The primary bow is caufed by two refractions and one reflection, and all the rays are reflected from the fame point.

88. The

88. The ſecondary bow is cauſed by two refractions and two reflections, and all the rays from the firſt to the ſecond reflection go parallel.

Hence as ſome rays are loſt by refraction at every reflection, the primary bow is brighter than the ſecondary.

89. In the primary bow the leaſt refrangible rays are uppermoſt; and in the ſecondary bow the moſt refrangible are uppermoſt.

Hence the order of the colours in the two bows is inverted.

Red is the leaſt refrangible ray, and then orange, yellow, green, blue, indigo, violet; this therefore is the order of the colours from the upper to the under ſide of the primary bow, and from the under to the upper ſide of the ſecondary.

90. The bow appears circular, for the eye is in the vertex of a cone, in the ſurface of which all the drops lie which render the rays of any one colour efficacious.

The angle which the ſide of the cone makes with the axis is equal to the angle under the incident and emergent rays when efficacious; this angle is called the ſemidiameter of the bow, and may be computed, by firſt computing the angles of incidence and refraction when the rays are efficacious by prop. 80. and then prop. 78. will give the angle. In the primary bow, the angles for the leaſt and greateſt refrangible rays are 42°.2′. and 40°.17′.; and for the ſecondary, they are 50°.57′. and 54°.7′. The difference of theſe reſpective angles would be the breadth of each bow if the ſun were a point; but as it is not, the breadth will be increaſed by the breadth of the ſun, or 32′. Hence the breadth of the primary bow is 2°.15′. and that of the ſecondary 3°.42′. The axis of the cone is directed to the ſun, and hence the ſun is directly oppoſite to the center of the bow.

91. The ſemidiameter of the bow is equal to the altitude of its higheſt point added to the altitude of the ſun.

Hence the altitude of the higheſt point is equal to the ſemidiameter of the bow diminiſhed by the altitude of the ſun. The primary bow therefore cannot be ſeen unleſs the ſun's altitude is leſs than 42°.2′. nor the ſecondary unleſs it is leſs than 54°.7′. When the ſun is in the horizon each bow appears a ſemicircle, becauſe the center then lies in the horizon; the bow therefore never can appear larger.

ON

ON THE EYE AND VISION.

92. The eye is formed of ſeveral mediums, which have the power of forming, upon optical principles, the images of objects before it upon the back part; the formation of which, in ſuch a ſituation, is the cauſe of viſion.

The eye is perfectly globular, except that the fore part is a little more convex than the reſt. It conſiſts of three mediums, or humours; that in the front is a tranſparent fluid like water, and is therefore called the *aqueous* humour; the next is called the *cryſtalline* humour, and is an hard ſubſtance like the white of an egg boiled, and in the form of a double convex lens having its back ſurface of the greateſt curvature; the hindermoſt is called the *vitreous* humour, and is ſomewhat like to ſoft jelly. The whole globe, except the fore part, is ſurrounded by three coats; the outermoſt is called the *ſclerotica*; the next the *choroides*, and the innermoſt the *retina*. The coat of the more prominent part before is called the *cornea*, being like horn, and is perfectly tranſparent. Adjoining to the choroides, on the front of the eye, and in the aqueous humour, is an opaque membrane called the *uvea*, in the middle of which there is a hole, called the *pupil*, for the admiſſion of light; this the eye has the power of contracting or enlarging for the admiſſion of leſs or more light, as the circumſtances of viſion may require. The cryſtalline humour is ſuſpended by a muſcle, called the *proceſſus ciliares*, and ſometimes the *ligamentum ciliare*. The ſeveral coats and ſurfaces of the humours are ſo ſituated as to have one ſtraight line, called the *axis* of the eye, perpendicular to them all. At the bottom of the eye, a little towards the noſe, there is a nerve which goes to the brain, called the *optic* nerve. This is a continuation of the retina. The nerves from each eye meet before they come to the brain.

Now the refractive powers of the aqueous and vitreous humours have been found by experiment to be about the ſame as common water, and that of the cryſtalline is a little greater: that is, the ſine of incidence to refraction out of air into the aqueous humour is as 4 : 3, out of the aqueous into the cryſtalline as 13 : 12, and out of the cryſtalline into the vitreous as 12 : 13. Hence the aqueous and the vitrious humours being ſuppoſed to have the ſame refractive power, may be conceived to form one medium in which the cryſtalline humour is ſituated; the rays are therefore firſt refracted at the cornea into this medium and are made to converge; and then falling upon the cryſtalline humour, or convex lens, are made to converge more, and come to a focus at the bottom of the eye; the whole may therefore be conſidered as a kind of compound lens. That the pictures of all objects are formed upon the bottom of the eye appears from hence, that if the ſclerotica at the bottom of the eye be taken off, the pictures of the objects before the eye will appear on the bottom.

A table

A table of the dimenfions of the human eye at a medium.

	in.
Diameter from the cornea to the choroides - - -	,95
Radius of the cornea - - - -	,335
Diftance of the cornea from the firft furface of the cryftalline	,106
Radius of the firft furface of the cryftalline - -	,331
Radius of the back furface of the cryftalline - -	,25
Thicknefs of the cryftalline - - -	,373

93. Having given the focus of incidence of rays upon the eye, the refraction of the mediums, and the radii of the cornea and each furface of the cryftalline humour, the focus after refraction may be found.

For the focus of rays refracted at the cornea may be found by prop. 38. and that focus is the focus of rays upon the cryftalline lens; hence by prop. 44. the focus after refraction by the lens may be found.

If vifion arife from the formation of the image upon the retina, or, as fome imagine, upon the choroides, it is manifeft that a different conformation of the eye is neceffary for diftinct vifion at different diftances. Some think it is a change in the length of the eye; others a change in the figure or pofition of the cryftalline humour; others that it is a change in the cornea. Any of thefe changes would produce the effect, and fufficient experiments have not yet been made to determine with certainty which is the true opinion. As the rays fuffer a greater refraction at the cornea, than they do afterwards, it is manifeft that a lefs change in the radius of the cornea will effect the bufinefs, than will fuffice in any other part of the eye.

94. A long fighted perfon muft ufe a convex lens to fee a near object diftinctly; and a fhort fighted perfon muft ufe a concave lens to fee a diftant object diftinctly.

A long fighted perfon has either the cornea or cryftalline humour, or both, too *flat*, and therefore the image of a near object would be formed beyond the bottom of the eye; this may be corrected by a convex lens, which makes rays converge more, and thereby form the image on the bottom of the eye. But a fhort fighted perfon has them too *fpherical*, and therefore the image is formed before the rays come to the bottom of the eye; to correct this, and form the image on the bottom, a concave lens muft be ufed, which makes rays diverge. If m be the diftance at which either perfon can fee diftinctly with the naked eye, and n the diftance at which he wants to fee with the glafs, then $\frac{mn}{m \backsim n}$ is the focal length of the glafs. Thofe who ufe glaffes fhould

have

have them as accurately as possible adapted to the eye, otherwise they will strain the eye. It is a common opinion that looking through glasses is detrimental to the eye; but this is so far from being true, that, to preserve the eye, those who want them ought to use them, otherwise the eye will continually be subject to be strained by endeavouring to see objects distinctly. Also opticians, who are continually examining telescopes, find no inconvenience from it.

95. The vibrations communicated by the optic nerve to the brain, from the impulses of the rays of light upon the bottom of the eye, are supposed to cause the sense of seeing.

This is the opinion of Sir I. Newton. He supposes also that the several sorts of rays make different vibrations, which accordingly excite sensations of different colours, much after the same manner that different vibrations of air excite sensations of different sounds. The flashes of lightning sometimes perceived by a blow upon the eye, or the colours from pressing the eye sideways, are probably owing to the same kind of vibrations being excited, as by the impressions of light. Such phænomena as these tend to confirm the hypothesis, that vision is caused by some motion excited in the optic nerve, by the impulse of light. The ideas we have from the impressions of light remain for a small time, as is manifest from the phænomenon of a burning coal appearing like a ring of fire, when whirled swiftly round. The stronger the light is the longer the sensation remains, as appears from looking at the sun, in which case the sensation will continue some miuutes. All these circumstances render it very probable that the sensation arises from a vibrating motion.

Sir I. Newton supposes that every point of the retina of one eye, hath its corresponding point on the other; from which two slender pipes filled with a liquid go along the optic nerve, and meeting before they come to the brain, their joint effect produce but one sensation. Hence if an eye be distorted, objects appear double, because corresponding points of the image do not fall upon these corresponding points of the retina. When we look directly at an object, the axis of both our eyes are directed to it, and the corresponding points of each image agree with those of the retina, and the object therefore appears single; but at the same time any other object in the same line either nearer or further off appears double, because corresponding points of each image do not, in this case, fall on those of the retina. If you shut one eye, the object not looked at directly then appears single. In a matter however of so much uncertainty, it is no wonder that different authors have invented different hypotheses.

When the image of an object falls upon the optic nerve, the object becomes invisible to that eye. Hence an object cannot become invisible to both eyes at the same time, because the image cannot fall upon the optic nerve of each eye at the same time. An object seen with both eyes appears about $\frac{1}{10}$ or $\frac{1}{12}$ brighter than with one eye.

The

The angle ſubtended by the leaſt viſible object cannot be very accurately aſcertained, as it depends upon the colour of the object, and the ground upon which it may be ſeen; it depends alſo upon the eye. Mr. HARRIS thinks the leaſt angle for any object to be about 40″; and at a medium about 2′.

To the generality of eyes the neareſt diſtance of diſtinct viſion is about 7 or 8 inches. Hence if we aſſume 7 inches for that diſtance, and 2 minutes for the leaſt viſible angle, a globular object of leſs than about $\frac{1}{250}$th part of an inch cannot be ſeen. See HARRIS's Optics upon theſe ſubjects.

96. The eye, as to ſenſe, corrects the different refrangibility of the rays of light.

For objects ſeen by the naked eye are not tinged with the priſmatic colours. EULER ſuppoſed the eye to be perfectly achromatic. Dr. MASKELYNE in the *Phil. Tranſ.* 1789, has examined this point, and taking the dimenſions of the eye from Mr. PETIT, and the refractive powers of the different mediums from Mr. HAUKSBEE, has computed the diameter of the circle of aberration upon the retina, and found it to be ,002667 of an inch; a quantity too ſmall to be perceived. He thinks ſome ſuch an angle of aberration as this is neceſſary in order to account for the ſenſible diameters of ſome of the fixed ſtars.

ASTRONOMY.

DEFINITIONS.

1. A *GREAT* circle of a ſphere is that whoſe plane paſſes through its center; and a *ſmall* circle is that whoſe plane does not paſs through its center.

2. A diameter of a ſphere perpendicular to any great circle, is called the *axis* of that circle, and the extremeties of the diameter, are called its *poles*.

Hence the pole of a great circle is 90° from every point of it upon the ſurface of the ſphere; but as the axis is perpendicular to the circle when it is perpendicular to any two radii, therefore a point on the ſurface of a ſphere 90°. diſtant from any two points of a great circle will be the pole.

3. All angular diſtances on the ſurface of a ſphere to an eye at the center, are meaſured by the arcs of great circles; for then being arcs to equal radii, they will be as the angles.

Hence all triangles formed upon the ſurface of a ſphere for the ſolution of ſpherical problems, muſt be formed by the arcs of great circles.

4. All great circles muſt biſect each other; for paſſing through the center of the ſphere their common ſection muſt be a diameter, which biſects all circles.

5. Secondaries to a great circle are great circles which paſs through its poles.

Cor. 1. Hence ſecondaries muſt be perpendicular to their great circle; for if one line be perpendicular to a plane, any plane paſſing through that line will alſo be perpendicular to it; hence as the axis of the great circle is perpendicular to it, and is the common diameter to all the ſecondaries, they muſt all be perpendicular to the great circle. Hence alſo every ſecondary muſt biſect its great circle, and every ſmall circle parallel to it; for the plane of the ſecondary paſſes through not only the center of the great circle, but alſo of the ſmall circles parallel to it.

Cor. 2. Hence a great circle paſſing through the poles of two great circles, muſt be perpendicular to each; and, vice verſâ, a great circle perpendicular to two other great circles muſt paſs through their poles.

6. A circle appears a ſtraight line to an eye in its plane; hence in the repreſentation of the ſurface of a ſphere upon a plane, thoſe circles whoſe planes paſs through the eye are repreſented by ſtraight lines.

7. The

7. The angle formed by two great circles on the ſurface of a ſphere is equal to the angle formed by the planes of the circles; and is meaſured by the arc of a great circle intercepted between them deſcribed about the interſection of the circles as a pole.

For let C (fig. 7.) be the center of the ſphere, PQE, PRE two great circles; draw the tangents Px, Pz, then the angle xPz = the angle formed by the two circles; and as theſe tangents are perpendicular to the common interſection PCE, the angle between them is equal to the angle between the planes, by Eu. B. 11. def. 6. Now draw CQ, CR perpendicular to PCE; then the angle QCR is the angle between the planes, and therefore equal to the angle formed by the two circles, and this angle is meaſured by the arc QR of a great circle, which arc has for its pole the point P by def. 2. becauſe PQ, PR are each 90°.

8. If at the interſection of two great circles as a pole, a great circle be deſcribed, and alſo a ſmall circle parallel to it; the arcs of the great and ſmall circles intercepted between the two great circles contain the ſame number of degrees.

For draw AB, AD perpendicular to PCE, then as AB, AD are parallel to CQ, CR, the plane ABD is parallel to the plane QCR, and therefore the ſmall circle BD of which A is the center is parallel to the great circle QR; and as each angle BAD, QCR, meaſures the inclination of the planes they muſt be equal, and conſequently the arcs BD, QR contain the ſame number of degrees. Hence the arc of ſuch a ſmall circle meaſures the angle at the pole between the two great circles. Alſo $QR : BD :: QC : BA ::$ radius : coſ. BQ.

Cor. Hence QR is the greateſt diſtance between the two circles; and if from R, a point 90°. from P, a great circle QR be drawn perpendicular to PQ, the arc RQ is the meaſure of the angle at P.

9. The *axis* of the earth is that diameter about which it performs its diurnal motion; and the extremities of this diameter, are called its *poles*.

10. The *terreſtrial equator* is a great circle of the earth perpendicular to its axis. Hence the axis and poles of the earth are the axis and poles of its equator. That half of the earth which lies on the ſide of the equator which we inhabit, is called *north*, and the other *ſouth*; and the poles, are called the north and ſouth poles.

11. The *latitude* of a place on the earth's ſurface, is an arc meaſured from the equator upon a ſecondary to it.

12. The *longitude* of a place on the earth's ſurface, is the arc upon the equator between a ſecondary to it paſſing through the place, and another ſecondary paſſing through any other place from which you begin to meaſure.

13. If the plane of the terreſtrial equator be produced to the ſphere of the fixed ſtars, it marks out a circle, called the *celeſtial equator*; and if the axis of the earth be produced in like manner, the points in the heavens to which it is produced, are called poles, being the poles of the celeſtial equator. The ſtar neareſt to each pole, is called the *pole* ſtar.

14. Secondaries to the celeſtial equator, are called circles of declination; becauſe the declination of an heavenly body is its angular diſ-

tance from the equator meaſured upon a ſecondary to it. Of theſe, 24, which divide the equator into equal parts, each containing 15°, are called *hour* circles.

15. Small circles parallel to the equator, are called *parallels of declination.*

16. The *ſenſible* horizon, is that circle in the heavens which bounds the ſpectator's view. The *rational* horizon, is a great circle in the heavens paſſing through the earth's center parallel to the ſenſible horizon.

17. If the radius of the earth to the place where the ſpectator ſtands be produced both ways to the heavens, that point vertical to him, is called the *zenith,* and the oppoſite point the *nadir.*

Hence the zenith and nadir are the poles of the rational horizon.

18. Secondaries to the horizon, are called *vertical* circles, becauſe they are perpendicular to the horizon, by def. 5. cor. 1.; on theſe circles therefore the altitude of an heavenly body may be meaſured.

19. A ſecondary common to the equator and horizon, and which therefore paſſes through the poles of each, by def. 5. cor. 2. is called the *meridian.*

20. The meridian of any place divides the heavens into two hemiſpheres, one of which is called the *eaſtern,* and the other the *weſtern* hemiſphere.

21. To a ſpectator on the north ſide of the equator, that direction which paſſes through the north pole, is called *north,* and the oppoſite direction *ſouth*; hence the meridian which paſſes through the zenith of the ſpectator and through the poles, muſt cut the horizon in the *north* and *ſouth* points.

22. A vertical circle which cuts the meridian of any place at right angles, is called the *prime* vertical; and the points where it cuts the horizon, are called the *eaſt* and *weſt* points.

Hence the eaſt and weſt points are 90°. diſtant from the north and ſouth. Theſe four, are called the *cardinal* points.

23. The *azimuth* of an heavenly body is its diſtance on the horizon, when referred to it by a ſecondary, from the north or ſouth points. The *amplitude* is its diſtance from the eaſt or weſt.

24. Small circles parallel to the horizon are called *almicanthars.*

25. The *ecliptic* is that great circle in the heavens, which the ſun appears to deſcribe in the courſe of a year.

26. The ecliptic and equator being great circles, the points where they biſect each other are called the *equinoctial* points.

27. The ecliptic is divided into 12 equal parts, called *ſigns*; Aries ♈, Taurus ♉, Gemini ♊, Cancer ♋, Leo ♌, Virgo ♍, Libra ♎, Scorpio ♏, Sagittarius ♐, Capricornus ♑, Aquarius, ♒, Piſces ♓. The order of theſe is according to the motion of the ſun, The firſt point of Aries coincides with one of the equinoctial points, and the firſt point of Libra with the other.

28. The motion of the heavenly bodies according to the order of the ſigns, is called *direct,* or *in conſequentia*; and the motion in the contrary direction, is called *retrograde,* or *in antecedentia.*

The *real* motion of all the planets is according to the order of the ſigns, but their *apparent* motion is ſometimes in a contrary direction.

29. The

29. The *right aſcenſion* of a body is an arc of the equator between the firſt point of Aries, and a declination circle paſſing through the body, meaſured according to the order of the ſigns.

30. The *oblique aſcenſion*, is the diſtance from the firſt point of Aries, to that point of the equator which riſes with any body, meaſured according to the order of the ſigns.

31. The *aſcenſional difference*, is the difference between the *right* and *oblique* aſcenſion.

32. The *longitude* of an heavenly body, is an arc of the ecliptic between the firſt point of Aries, and a ſecondary to the ecliptic paſſing through the body, meaſured according to the order of the ſigns.

33. The *latitude* of an heavenly body, is its angular diſtance from the ecliptic, meaſured upon a ſecondary to it drawn through the body. If that angle be ſeen from the earth, it is called the *geocentric* latitude, but as ſeen from the ſun, it is called the *heliocentric* latitude.

34. The *equinoctial colure*, is a ſecondary to the equator paſſing through the equinoctial points: the *ſolſtitial colure*, is a ſecondary common to the ecliptic and equator. Hence the ſolſtitial colure paſſes through the poles of the equator and ecliptic, by def. 5.

35. The *tropics* are two parallels of declination touching the ecliptic. One touching it at the beginning of Cancer, is called the tropic of Cancer; and the other touching it at the beginning of Capricorn, is called the tropic of Capricorn.

36. The *arctic* and *antarctic* circles are two parallels of declination, the diſtance of which from the two poles, is equal to the diſtance of the tropics from the equator.

37. A body is in *conjunction* with the ſun when it has the *ſame* longitude; and in *oppoſition*, when the difference of their longitudes is 180°. The conjunction is marked thus ☌, and the oppoſition thus ☍.

38. The *elongation* of a body is its angular diſtance from the ſun ſeen from the earth.

39. The *diurnal parallax* is the difference between the apparent places of the bodies in our ſyſtem when referred to the fixed ſtars, ſeen from the center and ſurface of the earth.

40. The *argument*, is a term uſed to denote any quantity, by which another required quantity may be found. For example, the argument of that part of the equation of time, which ariſes from the unequal angular motion of the earth in her orbit about the ſun, is the ſun's anomaly, becauſe the equation depends entirely upon the anomaly, and the latter being given, the former is immediately found. The argument of a planet's latitude is its diſtance from the node, becauſe upon this the latitude depends.

41. The *nodes*, are the points where the orbits of the primary planets cut the ecliptic, and where the orbits of the ſecondaries cut the orbits of their primaries. That node is called *aſcending* where the planet paſs from the ſouth to the north ſide of the ecliptic; and the other is called the *deſcending* node. The aſcending node is marked thus ☊, and the deſcending thus ☋.

42. The angle of *commutation*, is the angle at a planet formed by two lines one drawn to the earth and the other to the ſun; hence this angle

angle is the difference of the places of the planet ſeen from the earth and ſun.

43. The angle of *poſition*, is the angle at an heavenly body formed by two great circles, one paſſing through the pole of the equator, and the other the pole of the ecliptic.

Characters uſed for the Sun, Moon, Planets: the Sun ☉; the Moon ☾; Mercury ☿; Venus ♀; the Earth ⊖; Mars ♂; Jupiter ♃; Saturn ♄; the Georgium Sidus HL.

ON THE APPARENT MOTIONS OF THE HEAVENLY BODIES, THE DOCTRINE OF THE SPHERE, AND PRINCIPLES OF DIALLING.

1. The ſun appears to deſcribe the ecliptic in 365*d*. 6*h*. 9′. 10″,37; in which time it is found to be only twice in the equator. This is called a *ſidereal* year.

Cor. Hence the ecliptic is inclined to the equator.

The time the ſun deſcribes the ecliptic is found by taking the difference between its longitude and that of any fixed ſtar, and then obſerving when that difference becomes the ſame again; and the interval of thoſe times gives the time in which the ſun appears to make a complete revolution in the heavens. If 360° be divided by 365*d*. 6*h*. 9′. 10″,37 it gives 59′.8″, the ſpace the ſun *would* deſcribe in one day if all the days were of the ſame length, or the ſpace deſcribed in a *mean* ſolar day. Let a clock be adjuſted to go 24 hours in a mean ſolar day; then as the mean increaſe of the ſun's right aſcenſion in 24 hours is 59′.8″. the earth in a mean ſolar day muſt deſcribe about its axis 360°. 59′. 8″.; but when the earth has deſcribed 360°. the ſame fixed ſtar returns to the meridian; hence 360°. 59′. 8″. : 360°. :: 24 *h*. : 23 *h*. 56′. 4″. the length of a *ſidereal* day in *mean* ſolar time.

The interval of time from the ſun's leaving the firſt point of Aries, till his return to it, is 365*d*. 5*h*. 48′.48″. This is called a *tropical* year.

The return of the ſun to the firſt point of Aries, before it has completed its revolution amongſt the fixed ſtars, ſhows that the interſection of the equator with the ecliptic has a retrograde motion, called the *preceſſion of the equinoxes*. By comparing the place of the firſt point of Aries as obſerved by the antient aſtronomers with its preſent ſituation, it appears that its motion in 100 years, is 1°. 23′. 40″.

2. All the heavenly bodies appear daily to deſcribe circles, coincident with, or parallel to, the equator.

This is a conſequence of the earth's rotation about its axis.

The

The motions of all the bodies in our ſyſtem are referred to the fixed ſtars, whoſe relative ſituations not being altered by our own motion, they are conceived as placed in the concave ſurface of a ſphere having the eye in the center.

3. Thoſe bodies which are on the ſame ſide of the equator with the ſpectator, continue longer above the horizon than below; thoſe on the contrary ſide continue longeſt below.

Hence when the ſun is on the ſame ſide of the equator with the ſpectator, the days are longer than the nights; when on the contrary ſide, the nights are longeſt. Hence the variety of ſeaſons ariſes from the inclination of the ecliptic to the equator.

As the orbits of the moon and planets are alſo inclined to the equator, a variation of the times of their continuance above and below the horizon will alſo take place.

4. If a ſpectator be in the equator, all the heavenly bodies continue as long above the horizon as below.

Hence to a ſpectator at the equator the days are always 12 *h.* long.

5. If a ſpectator be at the pole, all the fixed ſtars appear to deſcribe circles parallel to the horizon, the equator now coinciding with the horizon; therefore they never riſe and ſet.

6. To a ſpectator at the pole, the ſun appears above the horizon all the time he is on the ſame ſide of the equator with the ſpectator; and below the horizon all the time he is on the contrary ſide.

Hence to a ſpectator at the pole, there is half a year day, and half a year night.

7. In north latitude, thoſe bodies which have north declination riſe between the eaſt and north; and thoſe which have ſouth declination riſe between the eaſt and ſouth.

8. The

8. The altitude of the pole above the horizon is equal to the latitude of the place.

If in lat. 45°. a ſpectator travel 69,2 miles upon the meridian towards the north, the pole will be 1°. higher. Hence, ſuppoſing the earth to be a ſphere, its circumference would be 24912 miles, and radius 3964: but the figure of the earth is a ſpheroid, whoſe polar diameter : the equatorial :: 229 : 230, according to Sir I. NEWTON.

9. The latitude of a place may be found by obſerving the greateſt and leaſt altitude of a circumpolar ſtar, corrected for refraction, and taking half their ſum.

10. All thoſe ſtars which are not further from the pole than the latitude of the place, never ſet.

Theſe are called circumpolar ſtars.

11. If the declination of a fixed ſtar be known, the ſum or difference of its meridian altitude and its declination, according as the ſtar is on the contrary or ſame ſide of the equator with the ſpectator, will give the latitude of the place.

12. The altitude of that point of the equator which is upon the meridian, or the inclination of the equator to the horizon, is equal to the complement of the latitude.

13. The greateſt declination of the ſun is equal to the inclination of the ecliptic to the equator.

14. The inclination of the equator to the ecliptic, is equal to half the difference between the ſun's meridian altitudes on the longeſt and ſhorteſt days.

The inclination at this time is about 23°. 28′.; but it keeps gradually diminiſhing at the rate of about $\frac{1}{2}''$ in a year. This ariſes from the variation of the ecliptic, which is owing to the attraction of the planets upon

upon the earth, by which the plane of its orbit is continually varying. The whole variation however can never exceed much more than a degree. It will decreafe for a confiderable time, and then increafe again.

15. The angle a° contained between the meridian of any place, and the circle of declination paffing through the fun, turned into time at the rate of 15° for an hour, gives the time from apparent noon.

This is not accurate, becaufe the folar days are not all equal to 24 hours, but to $24h. \pm e$ the variation of the equation of time for that day, according as the equation is increafing or decreafing: hence to get the time more accurately, fay, $360^\circ : a^\circ :: 24h. \pm e$: the time. The time is here fuppofed to be *mean* folar time, meafured by a clock which goes 24 hours in a mean folar day. This quantity e is fometimes $30''$, and therefore if $a^\circ = 60^\circ$ the correction is $5''$. If extreme accuracy were alfo required, the change of declination muft alfo be confidered, which in the moon may be confiderable.

If Z (fig. 8.) be the zenith of any place, P the pole, S the fun; and thefe points be joined by the arcs of three great circles, then ZS, ZP, PS, are the complements of the fun's altitude, of the latitude and of the fun's declination refpectively; alfo the angle ZPS is the meafure of the time from apparent noon, and SZP is the azimuth from the north.

16. Given the latitude of the place, the fun's altitude and declination, to find the hour and azimuth.

By fpherical trig. fin. $SP \times$ fin. $ZP : \text{rad.}^2 ::$ fin. $\frac{1}{2} \times \overline{SZ+SP-PZ} \times$ fin. $\frac{1}{2} \times \overline{SZ+PZ-SP}$: $\overline{\text{fin.} \frac{1}{2} ZPS}^2$, hence ZPS is known, which converted into time at the rate of 15° for an hour, gives the time from apparent noon. Alfo fin. $SZ \times$ fin. $ZP : \text{rad.}^2 ::$ fin. $\frac{1}{2} \times \overline{SP+SZ-ZP} \times$ fin. $\frac{1}{2} \times \overline{SP+ZP-SZ}$: $\overline{\text{fin.} \frac{1}{2} SZP}^2$, hence the azimuth SZP from the north is known.

17. Given the latitude of a place, the altitude, right afcenfion and declination of a fixed ftar, to find the time.

See my *Practical Aftronomy*, pag. 53, where the reader will find the rule with an example; alfo an example to find the time by the fun. Thefe are the methods ufually practifed at fea for finding the time.

18. Given the latitude and ſun's declination, to find its altitude on the prime vertical, and the time.

In this caſe the angle Z is a right one; hence coſ. ZP : rad. :: coſ. SP : coſ. ZS, or, ſin. lat. : rad. :: ſin. dec. : ſin. of the altitude. Alſo rad. : cot. PS :: tan. PZ : coſ. ZPS, or rad. : tan. dec. :: cot. lat. : coſ. ZPS, which converted into time, gives the time from apparent noon.

19. Given the latitude and ſun's declination, to find the altitude at 6 o'clock.

Here the angle ZPS is a right one; hence rad. : coſ. ZP :: coſ. PS : coſ. ZS, or rad. : ſin. lat. :: ſin. dec. : ſin. of the altitude.

20. Given the latitude and ſun's declination, to find the time of its riſing, and azimuth at that time.

When one ſide of a ſpherical triangle $= 90°$, the triangle may be ſolved by the circular parts, juſt as when one angle is a right one, by taking the angles adjacent to the ſide of 90° and the complements of the other three parts for the circular parts; for if we conceive the ſupplemental triangle to be taken, it will have a right angle, and the ſine, coſine and tangent of any arc is the ſame as of the ſupplement, regard not being had to the ſigns of the two latter, which is here of no conſequence. This circumſtance is not, that I know of, taken notice of by any writers on ſpherical trigonometry. Hence, as $ZS = 90°$ in this caſe, rad : cot. SP :: cot. ZP : coſ. ZPS, or rad. : tan. dec. :: tan. lat. : coſ. of the hour angle from apparent noon. This ſuppoſes that the body is upon the rational horizon at the inſtant it appears; but upon account of refraction, bodies in the horizon appear 33′ higher than their true places; hence they become viſible when they are 33′ below the horizon. Alſo bodies in our ſyſtem are depreſſed by parallax; hence when ſuch bodies firſt appear, $ZS = 90° + 33' -$ hor. par. Alſo ſin. ZP : rad. :: coſ. SP : ſin. PZS, or coſ. lat. : rad. :: ſin. dec. : ſin. of azimuth from the north.

21. Given the latitude and ſun's declination, to find the time when twilight begins.

Twilight begins when the ſun is about 18° below the horizon; hence $ZS = 108°$; therefore ſin. $SP \times$ ſin. ZP : rad.^{2} :: ſin. $\frac{1}{2} \times \overline{108°. + SP - PZ} \times$ ſin. $\frac{1}{2} \times \overline{108°. + PZ - SP}$: $\overline{\text{ſin.} \frac{1}{2} ZPS}^2$, hence ZPS is known, which converted into time, gives the time from apparent noon. Twilight is cauſed by the refraction of the ſun's rays by the atmoſphere. It is obſerved that the diſtance of the ſun below the horizon when twilight ends in the evening, is greater than its diſtance from the horizon in the morning when it begins; it is longer alſo in ſummer than in winter.

22. As

22. As the apparent diurnal motion of the ſun about the axis of the earth is at the rate of 15° in an hour, if the earth were tranſparent and the axis opaque, its ſhadow would revolve at the ſame rate, being always projected into the meridian oppoſite to the ſun.

23. If we conceive a plane paſſing through the center of the earth, coinciding with the rational horizon of any place, and right lines be drawn from the center to the points where the hour circles cut that plane, they will repreſent the hour lines on an horizontal dial for that place.

In every dial the gnomon, when fixed, is parallel to the earth's axis, and on account of the ſun's great diſtance compared with the radius of the earth, the apparent motion of the ſun may be conceived to be the ſame about the gnomon as about the earth's axis, and therefore an horizontal dial may be conſtructed ſimilar to the conſtruction in the propoſition. Now when the ſun is in the meridian, the 12 o'clock hour circle is perpendicular to the plane, and the arc from the pole to the plane is equal to the latitude of the place, and the 1 o'clock hour circle makes an angle at the pole with it of 15° and forms the hypothenuſe of a right angled triangle to the above perpendicular, and the baſe is the arc meaſuring the angle between the 12 and 1 o'clock line; to find which we have, by ſpher. trig. rad. : ſin. lat. :: tan. 15° : tan. of the hour angle between 12 and 1 o'clock. If inſtead of 15° we put 30°, 45°, &c. we ſhall get the angles between the 12 and 2, 3, &c. o'clock lines.

24. If we conceive a plane paſſing through the center of the earth perpendicular both to the horizon and meridian, and on the ſouth ſide lines be drawn from the center to the points where the hour circles cut that plane, they will repreſent the hour lines on a vertical ſouth dial.

Hence a vertical ſouth dial may be conſtructed in a manner ſimilar to this conſtruction. In this caſe, the arc of the meridian from the pole to the plane, is equal to the complement of latitude; hence, for the ſame reaſon as before, rad. : coſ. lat. :: tan. 15° : tan. of the hour angle between 12 and 1 o'clock. In like manner, as before we get

the other hour angles. Upon the ſame principles, the hour angles may be calculated for any other plane.

The general principles of dialling may alſo be explained in the following manner. Inſert an axis in a cylinder; divide the circumference of one end into 24 equal parts, and draw lines from them to the center, and theſe lines will be the hour lines for a polar dial. From theſe points of diviſion, draw lines upon the ſurface of the cylinder parallel to the axis, and cutting the cylinder through by any ſection, draw lines from theſe parallel lines to the center of the ſection, and placing the axis of the cylinder parallel to the earth's axis, you have a dial for that plane.

ON PARALLAX AND REFRACTION.

25. Every body appears elevated, by the refraction of the atmoſphere, above its true place.

This follows from the common principles of Optics. Tab. 1. gives the refraction at all altitudes.

26. Every body appears depreſſed by parallax below its true place in a vertical circle; and the ſine of the parallax varies as the ſine of the zenith diſtance directly, and the diſtance of the body from the center of the earth inverſely.

The horizontal parallax of the ſun, as determined from the tranſit of venus, is 8″,7; hence if we take the radius of the earth = 3964 miles, we have ſin. 8″,7 : rad. :: 3964 : 94000474 miles, the ſun's diſtance. The real diſtance of the ſun being thus known, and their relative diſtances from their periodic times, the real diſtances of all the planets will be known.

The diſtance of the fixed ſtars is ſo great that they have no diurnal parallax. It appears alſo that they have no annual parallax.

When the altitude of a body is obſerved, it muſt be corrected by parallax and refraction, adding the former, and ſubtracting the latter, in order to get the true altitude, or the altitude above the rational horizon at the center of the earth.

ON PRACTICAL ASTRONOMY, AND THE INSTRUMENTS FOR THAT PURPOSE.

27. The VERNIER is a graduated index moveable againſt the arc of a quadrant, or any graduated

ated line, in order to ſubdivide it to a greater degree of accuracy than could be done by an actual ſubdiviſion.

The principal is this: if equal arcs A of the quadrant and its index be divided, one into n and the other into $n+1$ equal parts, the difference of the lengths of each diviſion $= \frac{A}{n \times \overline{n+1}}$. If $A = 7°$, and each degree be divided into 3 equal parts, then A is divided into 21 equal parts; and if an arc A of the index be divided into 20 equal parts, the difference of each diviſion $= \frac{70}{20 \times 21} = 1'$. Hence when any two diviſions coincide, the diſtance of the next two $= 1'$, of the next two $= 2'$, of the next two $= 3'$, &c. This method of ſubdiviſion is applied to moſt quadrants.

28. The TRANSIT TELESCOPE is a teleſcope moveable about an horizontal axis, and ſo adjuſted as to make its line of collimation deſcribe a great circle paſſing through the pole and zenith, or the meridian of the plane.

The line of collimation is the line joining the center of the object glaſs and the center of the croſs wires in its principal focus. One of the croſs wires paſſing through the center is perpendicular to the horizon, and conſequently it coincides with the meridian. Hence when any body comes to this wire it is in the meridian. The uſe of this inſtrument is to take the right aſcenſions of the heavenly bodies, and to correct the going of the clock.

A ſidereal day is the interval between the two ſucceſſive paſſages of a fixed ſtar over the meridian. A *ſolar* day is the interval between the two ſucceſſive paſſages of the ſun's center over the meridian; theſe days are not all equal. If we conceive the year to be divided into the ſame number of days of equal lengths, ſuch a day, is called a *mean* ſolar day. A clock adjuſted to go 24 hours in a *ſidereal* day, is ſaid to be adjuſted to *ſidereal* time. If it be adjuſted to go 24 hours in a mean ſolar day, it is ſaid to be adjuſted to *mean* ſolar time.

29. The interval of time between the two ſucceſſive paſſages of a fixed ſtar over the meridian : the interval between the paſſages of two fixed ſtars :: 360° : the difference of their right aſcenſions.

Hence if we know the right aſcenſion of one ſtar, we can find the right aſcenſion of all the others. The method of firſt finding the right aſcenſion

ascension of a star is explained in my *Practical Astronomy*, p. 86. If we thus compare the right ascension of a known fixed star with that of the sun, moon, or a planet, we shall get their right ascensions. A clock adjusted to sidereal time, is that by which we determine right ascensions; transit clocks in observatories are therefore thus adjusted.

30. If x be the difference of the sun's and a planet's motion in right ascension in 24 hours, reduced into time, t the difference of their right ascensions in time when the sun is on the meridian, then $\frac{24t}{24 \pm x}$ is the time from apparent noon when the planet is on the meridian, where the upper or lower sign prevails, according as the planet's or sun's motion is the greatest. If the planet be retrograde, x must be the sum of the motions of the sun and planet with the sign $+$.

As $\frac{24t}{24 \pm x} = t \pm \frac{tx}{24} + \frac{tx^2}{24^2} \pm$ &c. the two first terms will be sufficiently exact for all cases, except that of the moon, where it will be necessary to take the next term. The values of t and x may be put down in decimals, by tab. 2. Apparent noon is the time when the sun's center is on the meridian.

Ex. On July 1, 1767, the sun's AR when on the meridian of Greenwich, was $6h.40'.25''$, and its daily increase $4'.8''$; also the moon's AR, was $10h.36'.8''$, and its daily increase $42'.28''$, to find the time of the moon's passage over the meridian. Here $t = 10h.36'.8'' - 6h.40'.25'' = 3h.55'.43'' = 3{,}9285$, also $x = 42'.28'' - 4'.8'' = 38'.20'' = {,}6388$; hence $\frac{tx}{24} = 6'.16''$, $\frac{tx^2}{24^2} = 10''$; therefore $3h.55'.43'' + 6'.16'' + 10''. = 4h.2'.9''$. the time from apparent noon. If the equation of time be applied, it gives the time by the clock.

31. The ASTRONOMICAL QUADARNT is an instrument for measuring the altitudes of the heavenly bodies above the horizon.

Some quadrants turn upon a vertical axis, by which the altitudes of bodies in any situation may be measured; others, called *mural* quadrants, are fixed against a wall with their plane in the meridian, with which you can only measure meridian altitudes. Instead of a quadrant for

for taking altitudes, Mr. RAMSDEN has invented a new inſtrument, called a *circular inſtrument*, which has many advantages over that of the quadrant. A deſcription of this may be ſeen in my *Practical Aſtronomy*. After the altitude is taken it muſt be corrected for parallax and refraction, by which you get the true altitude above the rational horizon.

32. The latitude of the place and the meridian altitude of a body being known, its declination will be known.

33. The right aſcenſion and declination being known, the place of the body in the heavens is known.

34. Given the right aſcenſion and declination of an heavenly body, its latitude and longitude may be computed.

The beſt rule for this purpoſe is that given by Dr. MASKELYNE, which the reader may ſee in my *Practical Aſtronomy*, pag. 113.

The foundation of all aſtronomy is to determine the ſituation of the fixed ſtars, in order to refer the places of other bodies to them from time to time, and from thence to determine their proper motions.

35. If equal altitudes of an heavenly body be taken on different ſides of the meridian, the middle point of time between will give the time when the body is upon the meridian, if it have not changed its declination.

The correction for the variation of declination may be ſeen in my *Practical Aſtronomy*, pag. 44. By this means the time when any body comes to the meridian may be found; and when applied to the ſun or a fixed ſtar, the rate at which a clock, adjuſted to mean ſolar or ſidereal time, gains or loſes may be determined.

36. If a ſmall error m be made in taking the altitude of the ſun or a ſtar; the correſponding error of time will be equal to $m \times \frac{\text{rad.}^2}{\text{coſ.lat.} \times \text{ſin.azim.}}$; the time being found by prop. 16.

The

The inveſtigation of this may be ſeen in my *Practical Aſtronomy*, pag. 42.

Hence when the latitude is given the error in time is the leaſt upon the prime vertical, and is independent of the declination. To find the time therefore from an obſerved altitude of the ſun or a ſtar, it ſhould be taken upon the prime vertical, as being ſubject to the leaſt error. In lat. 52°.12′, if the error in altitude at an azimuth 44°.22′ be 1′, the error in time $= 1' \times \frac{1^2}{,613 \times ,699} = 2',334$ of a degree $=$ 9″,336 the error of time.

37. The EQUATORIAL INSTRUMENT conſiſts of three circles, the azimuth circle, the equatorial circle and the declination circle; the firſt may be adjuſted parallel to the horizon, the ſecond parallel to the equator and the third perpendicular to the equator.

The declination circle has a teleſcope applied to it, whoſe line of collimation is parallel to that circle.

The uſes of this inſtrument are — to take the altitude of a body above the horizon — to determine the poſition of the meridian — to determine the time of the day — to find a ſtar or planet in the day time — to find the right aſcenſion and declination of a ſtar, and to meaſure horizontal angles.

38. The EQUATORIAL SECTOR is an inſtrument for taking the difference of the right aſcenſions and declinations of ſtars.

There are various conſtructions of this inſtrument; Mr. GRAHAM conſtructed the firſt; afterwards Dr. MASRELYNE conſtructed one upon different principles, which had many advantages over that by Mr. GRAHAM.

39. The ZENITH SECTOR is an inſtrument conſtructed for the purpoſe of meaſuring ſmall angular diſtances from the zenith.

This inſtrument was invented by Dr. HOOK, in order to determine the annual parallax of the fixed ſtars, as, upon account of the length of its radius, it is capable of meaſuring ſmall angles with greater accuracy than the quadrant. It was with this inſtrument, that Dr. BRADLEY made his two admirable diſcoveries of the *aberration of light* in the fixed ſtars, and the *nutation* of the earth's axis.

40. A

40. A MICROMETER is an inſtrument invented at firſt to meaſure the angular diſtances of ſuch bodies as appear in the field of view of a teleſcope at the ſame time, or the diameters of the ſun, moon or planets. But the uſe was afterwards extended to meaſure the diſtances of bodies more remote from each other.

41. HADLEY'S QUADRANT is an inſtrument for meaſuring the altitudes of bodies above the horizon, or the angular diſtance of any two bodies, whatever be their poſition.

As this inſtrument is conſtructed to meaſure the angular diſtance of any two bodies, and as it will do this even when the obſerver is ſubject to any unſteadineſs, it is extremely well adapted to find the longitude at ſea, by the moon's diſtance from the ſun or a fixed ſtar. With a good inſtrument the time found from taking the altitude of the ſun or a ſtar at land, may be depended upon to about 2″ in our latitude, if the altitude be taken on or near the prime vertical. Altitudes at land are taken by reflection from an artificial horizon. The ſurface of all fluids are horizontal; but as they are ſubject to be diſturbed by the wind, various contrivances have been invented to render reflecting planes horizontal by means of fluids. The altitude at ſea is found by the real horizon, allowing for the dip.

A very full deſcription of all theſe inſtruments, with an account of their uſes, may be ſeen in my *Practical Aſtronomy*.

42. A whirling table is an inſtrument to ſhow the doctrine of centripetal forces.

43. The eclipſareon is an inſtrument to ſhow all the phænomena of a ſolar eclipſe.

By this inſtrument the time and quantity of an eclipſe at any place, may be determined to a very conſiderable degree of accuracy.

44. The figure of the earth, ariſing from the centrifugal force of its parts in conſequence of its rotation about its axis, may be ſhown by the re-

volution of two braſs hoops placed at right angles to each other, and made to revolve about a common diameter.

ON THE INEQUALITY OF SOLAR DAYS.

45. By comparing the right aſcenſion of the ſun every day at noon with that of a fixed ſtar, it appears that the right aſcenſion of the ſun does not increaſe uniformly.

46. As the earth revolves uniformly about its axis in the ſame direction in which it revolves about the ſun, and the daily increaſe of the ſun's right aſcenſion is not uniform, the ſolar days, being equal to the time of the earth's rotation + the time of its deſcribing an arc equal to the increaſe of the ſun's right aſcenſion in a ſolar day, are not all equal.

47. If the motion of the earth in its orbit, or the apparent annual motion of the ſun, were uniform and in the equator, the ſolar days would be all equal.

For the right aſcenſion is meaſured upon the equator, and therefore in this caſe its increaſe would be uniform. Hence if we conceive an imaginary ſtar to move uniformly in the equator with the ſun's mean motion in right aſcenſion, the intervals of its tranſits over the meridian would be all equal. A clock therefore ſet to 12 when this ſtar is upon the meridian, and adjuſted to go 24 hours in that interval, would always agree with the ſtar, that is, ſhow 12 at the time of the tranſit. Hence it is the ſame thing, whether we compare ſolar time with the time meaſured by this ſtar or by the clock.

48. The ſun does not move in the equator, nor does it move uniformly; therefore the ſolar days are not equal. Hence the inequality of ſolar days, ariſes from the inclination of the equator to the ecliptic,

ecliptic, and the unequal angular velocity of the earth in its orbit.

As the ſolar days are not all equal, a clock adjuſted as above deſcribed, would not always ſhow 12 when the ſun is on the meridian. The time from 12 when the ſun is on the meridian, is called the *equation of time*.

The time by the ſun is called *apparent*, and that by the clock *true* or *mean* time.

The practical method of computing the equation of time not being given in any books of aſtronomy, we ſhall here give the inveſtigation, with an example.

Let $APLS$ (fig. 9.) be the ecliptic, ALv the equator, A the firſt point of Aries, P the ſun's apogee, S the place of the ſun, draw Sv, Pw perpendicular to Av, and take $Ln = LP$. When the ſun ſets out at P, let the imaginary ſtar ſet out at n with the ſun's mean motion in right aſcenſion or longitude, or at the rate of $59'.8''$ in a day, and when n paſſes the meridian let the clock be adjuſted to 12, in which caſe nw is the equation of time; theſe are the correſponding poſitions of the clock and ſun as aſſumed by aſtronomers. Take $nm = Ps$, and when the ſtar comes to m, the place of the ſun, if it moved uniformly with its mean motion, would be at s, but at that time let S be the place of the ſun. Now as the clock is adjuſted to 12 at the time the ſtar at n paſſes the meridian, and as at that time the ſun's true place in the equator is v, mv is the equation of time. Now s is the ſun's mean place, and as $An = AP$ and $nm = Ps$, $\therefore$ $Am = APs$, conſequently $mv = Av - Am = Av - APs$. Now let a be the mean equinox, and draw az perpendicular to AL; then $Am = Az + zm = Aa \times \text{cof.}\ aAz + zm = \frac{11}{12} Aa + zm$; hence $mv = Av - \frac{11}{12} Aa - zm$; now Av is the ſun's true right aſcenſion, zm is the mean right aſcenſion or mean longitude, and Az is the equation of the equinoxes in right aſcenſion; hence the equation of time is equal to *the difference of the ſun's true right aſcenſion, and its mean longitude corrected by the equation of the equinoxes in right aſcenſion*. When Am is *leſs* than Av, true time precedes apparent, and when *greater*, apparent time precedes true; becauſe the earth turns about its axis in the direction Av, or order of right aſcenſion, that body whoſe right aſcenſion is leaſt muſt come to the meridian firſt. This rule for computing the equation of time was firſt given by Dr. MASKELYNE in the *Phil. Tranſ.* 1764.

As a meridian of the earth, when it leaves m, returns to it again in 24 hours, it may be conſidered, when it leaves that point, as approaching a point at that time 360° from it, and at which it arrives in 24 hours. Hence the relative velocity with which a meridian accedes to or recedes from m is at the rate of 15° in an hour. Hence when the meridian paſſes through v, the arc vm reduced into time at the rate of 15° in an hour, gives the equation of time at that inſtant. Hence the equation of time is computed for the inſtant of apparent noon. Now the time of apparent noon in mean ſolar time, for which we compute, can only be known by knowing the equation of time. To compute therefore the equation on any day, we muſt aſſume the equation

the ſame as on that day four years before, from which it will differ but very little, and it will give the time of apparent noon, ſufficiently accurate for the purpoſe of computing the equation. If you do not know the equation four years before, compute the equation for noon mean time, and that will give apparent noon accurately enough.

Ex. To find the equation of time on July 1st. 1792, for the meridian of *Greenwich*, by *Mayer's* Tables.

The equation on July 1st. 1788, was 3'.28", to be added to apparent noon, to give the correſponding mean time; hence for July 1st. 1792, at 0*h*. 3'. 28" compute the true longitude.

	Mean Long. ☉	Long. ☉'s Ap.	N. 1.	N. 2.	N. 3.	N. 4.
Epoch for 1752.	9s.10°.50'.0",7	3s.9°.23'.46"	241	227	123	478
Mean Mot. July 1	5. 29. 23. 16,2	33	163	456	312	27
3'	7,4					
28"	1,1					
Mean Long.	3.10.13.25,4	3. 9. 24. 19	404	683	435	505
Equ. Cen.	− 1. 37,1	3.10.13. 25,4				
Equ. ☽ I.	+ 4,5					
♃ II.	− 4,7	49. 6,4	Mean Anom.			
♀ III.	+ 3,65					
☊ IV.	− 0,6					
True Long,	3. 10. 11. 51,15					

With this true longitude and obliquity 23°.27'.48",4 of the ecliptic, the true right aſcenſion of the ſun is found to be 3s.11°.5'.41",25; alſo the equation of the equinoxes in longitude = − 0,6; hence

Mean Long.	3s.10°.13'.25",4
$\frac{11}{12}$ of − 0,6	− 0,55
Mean Long. cor.	3. 10. 13. 24,85
Tr. AR.	3. 11. 5. 41,25
Equ.	52. 16, 4

which converted into time gives 3'.29",1 for the true equation of time; which muſt be added to apparent to give true time, becauſe the true right aſcenſion is greater than the mean longitude.

ON THE SYSTEM OF THE PRIMARY PLANETS.

49. Mercury and Venus revolve about the ſun in orbits included within that of the earth's orbit.

For they have all the phænomena which bodies ſo revolving muſt neceſſarily have, that is, two conjunctions, one between the ſun and the earth,

earth, for they ſometimes appear to paſs over the ſun's diſc as black ſpots, and the other beyond the ſun, when they ſhine with full faces. Between theſe ſituations there is a certain elongation from the ſun which they never exceed. From inferior to ſuperior conjunction, Venus is obſerved to have all the phaſes of the moon from new to full. Mercury is alſo obſerved to have the ſame phaſes. The former conjunction is called *inferior* and the latter *ſuperior*. Theſe are called *inferior* planets.

50. If the orbits of theſe planets were circular, the diſtance of each from the ſun would be to the earth's diſtance, as the ſine of its greateſt elongation to radius.

The orbits are not circles but ellipſes, having the ſun in the focus; for upon that ſuppoſition their computed places are always found to agree with their obſerved places. Their greateſt elongations obſerved in different parts of their orbits are accordingly found to be different. That of Venus being greater than that of Mercury, Mercury muſt be nearer to the ſun than Venus.

51. An inferior planet is direct through ſuperior conjunction and retrograde through inferior, between which ſituations it is found to be ſtationary.

52. Venus is a morning ſtar from inferior to ſuperior conjunction; and an evening ſtar from ſuperior to inferior.

53. The earth revolves in an ellipſe about the ſun, having the ſun in one of the foci.

For the computations of the ſun's place upon this ſuppoſition, allowing for the diſturbing forces of the planets, are found to agree with obſervations.

54. Mars, Jupiter, Saturn and the Georgium Sidus revolve in orbits including that of the earth.

For they appear in oppoſition, from whence to conjunction they are full orbed, except Mars which is a little Gibbous in Quadratures. The Georgium Sidus was diſcovered by Dr. HERSCHEL in 1781. Theſe are called *ſuperior* planets.

55. A

55. A ſuperior planet is retrograde in oppoſition and direct in conjunction, between which ſituations it is obſerved to be ſtationary.

56. If a be the diſtance of a planet from the ſun, the earth's diſtance being unity, and x be the ſine of the angle of elongation at the earth when the planet is ſtationary, then $x = \frac{a}{\sqrt{a^2 + a + 1}}$, the orbits being ſuppoſed to be circular, and the ſun in the center.

57. When a planet is ſtationary, if $t =$ the tangent of its elongation, its diſtance a from the ſun $= \frac{1}{2}t^2 + t\sqrt{\frac{5}{4}}$.

Hence if the elongation of a planet be obſerved when it is ſtationary, its diſtance from the ſun may be found compared with the earth's diſtance, upon ſuppoſition that the orbits are circular. If the excentricity of the orbit be ſmall, we ſhall thus get a very near value of the diſtance.

Ex. In 1782, March 6*d*. 6*h*. 14′. 56″ the Georgium Sidus was ſtationary with 2*s*.28°.49′.27″ apparent longitude, and 15′. 53″ latitude; now the place of the ſun at that time was 11*s*.17°.37′; hence the difference of their longitudes was 3*s*.11°.12′.27″, conſequently the diſtance of the planet from the ſun was 101°.12, being the hypothenuſe of a right angled ſpherical triangle, whoſe baſe was 101°.12′.27″ and perpendicular 15′.53″; hence $t = 5{,}05$, conſequently $a = 18{,}4$ which is a near diſtance of the Georgium Sidus from the ſun, the diſtance of the earth from the ſun being unity. If we thus compute the diſtances of the ſuperior planets, it appears that the order of their diſtances from the ſun are Mars, Jupiter, Saturn, the Georgium Sidus. By computing their diſtances from the ſun in different parts of their orbits, it appears that they are not always at the ſame diſtance from the ſun; and by aſſuming an elliptic orbit having the ſun in one of the foci, their computed places are found to agree with obſervation.

58. To determine the periodic time of a planet.

Obſerve when a planet is in any point of its orbit, and after any number of revolutions obſerve when it comes to the ſame point again; then divide that interval of time by the number of revolutions and you get the time of one revolution. The obſervations of the ancient aſtronomers are here very uſeful; for as they have put down the places of the

the planets from their obſervations, by comparing them with the places obſerved now, we take in a very great number of revolutions, and therefore if we divide the interval of time by the number of revolutions, if a ſmall error be made in the whole time it will affect ſo much leſs the time of one revolution.

59. The ſquares of the periodic times of the planets, have the ſame proportion as the cubes of their mean diſtances.

This law was diſcovered by KEPLER, after having found their periodic times and relative diſtances. Afterwards Sir I. NEWTON proved, from the laws of gravity, that it muſt be ſo.

60. If the planets moved in circular orbits, the time from conjunction to conjunction, or from oppoſition to oppoſition of any two, would be equal to the product of their periodic times divided by their difference.

61. The viſible enlightened part of a planet varies as the verſed ſine of the exterior angle at the planet, of a triangle formed by joining the centers of the earth, ſun and planet.

62. The geocentric latitude of a planet and its elongation from the ſun being known, the heliocentric latitude and longitude may be computed, the ratio of the diſtances of the earth and planet from the ſun being known.

63. Two heliocentric latitudes of a planet, and the difference of longitudes being known, the place of the node and the inclination of the orbit may be computed.

The following table contains the relative diſtances, periodic times, places of the nodes, inclinations of the orbits to the ecliptic and places of the aphelia of the orbits, of all the planets, according to M. DE LA LANDE.

	Mean Dist.	Sidereal Revol.	Node in 1750.	Inclinat. 1780.	Aphelia 1750.
Mercury	38710	87*d*.23*h*.15′.44″	1*s*.15°.20′.43′	7°.0′.0″	8*s*.13°.33′.58″
Venus	72333	224. 16. 49. 11	2. 14. 26. 18	3.23.35	10. 7. 46. 42
Earth	100000	365. 6. 9. 12			3. 8. 39. 34
Mars	152369	686. 23. 30. 36	1. 17. 38. 38	1.51. 0	5. 1. 28. 14
Jupiter	520279	4332. 14. 27. 11	3. 7. 55. 32	1.18.56	6. 10. 21. 4
Saturn	954072	10759. 1. 51. 11	3. 21. 32. 22	2.29.50	8. 28. 9. 7
Geor. Sid.	1908180	83*ys*. 150*d*. 18*h*.	3. 12. 33. 31	0.46.20	11. 17. 6. 44

The nodes, inclinations of the orbits and aphelia have a ſmall motion.

Beſides theſe bodies which revolve about the ſun, there are others, called *comets*, which revolve in very excentric ellipſes. Theſe, as they approach the ſun, have generally tails which increaſe till they come to their perihelia, and then decreaſe again. They are inviſible for the greateſt part of their revolutions.

The diameters of the planets, Sun and Moon in Engliſh miles are, of the Georgium Sidus 33954, of Saturn 78236, of Jupiter 92414, of Mars 5195, of the Earth 7928, of Venus 7609, of Mercury 3189, of the Sun 877547, and of the Moon 2326.

The mean ſemidiameter of the ſun was here taken 16′.3″ according to the Nautical Almanac; and the mean ſemidiameter of the moon 16′.44″,5 according to M. DE LA LANDE.

The Earth revolves about her axis in 23*h*. 56′. 4″ mean ſolar time; Saturn in 12*h*.13¼′; Jupiter in 9*h*.56; Mars in 24*h*.40; Venus in 23*h*. 20′; and the Sun in 25*d*.10*h*.

The time of Saturn's rotation is computed from Dr. HERSCHEL's ratio of its diameters, which he makes about 11 : 10. The time of rotation of the other planets, or whether they do revolve about their axes or not, have not yet been determined.

ON THE MOON.

64. The moon revolves about the earth.

65. The orbit which the moon deſcribes about the earth is an ellipſe, having the earth in one of the foci.

For the computations of the moon's place upon this ſuppoſition, allowing for the diſturbing force of the ſun, agree with obſervation.

66. The enlightened part of the moon varies as the verſed ſine of its elongation from the ſun.

Hence, as the curve which divides the light from the dark part of the moon appears an ellipſe, its phaſes may at any time be delineated.

68. If

67. If the angular velocity of the moon about the earth were equal to the angular velocity about her axis, the ſame face would always be turned towards the earth.

68. The angular velocity of the moon about the earth is not uniform, but the angular velocity about her axis is uniform, therefore the ſame face is not always turned towards the earth.

69. The time of the moon's rotation about her axis is equal to the mean time in her orbit, becauſe we never ſee the oppoſite ſide of the moon.

The conſequence of the two laſt propoſitions is, that the moon ſometimes ſhows a little more of her eaſtern, and ſometimes of her weſtern limb, and this is called a *libration in longitude*.

70. The moon's axis is not perpendicular to the plane of her orbit, in conſequence of which, we ſee ſometimes one pole and ſometimes the other; this is called a *libration in latitude*.

The inclination of her axis to her orbit is 6°.49′.

71. In north latitudes when the moon's right aſcenſion is nothing, her orbit at the time of her riſing makes the leaſt angle with the horizon; therefore the difference of the times of riſing on two ſucceſſive nights is then the leaſt.

In September this happens when the moon is at the full, and this is called the *harveſt moon*. The ſame circumſtance takes place every month, but as it does not happen at the time of the full moon, it is not taken notice of. When her AR.=6*s* there is the greateſt difference of the times of riſing. Thoſe ſigns which riſe with the leaſt angle ſet with the greateſt, and the contrary; therefore when there is the leaſt difference in the times of riſing, there is the greateſt in ſetting, and the contrary.

The lunar months, and mean diſtances of the moon at different times of the year, are not all equal, owing to the action of the ſun upon it being different. The time of a mean ſidereal month was 27*d*:7*h*.43′.11½″

at the beginning of this century; but the mean motion is accelerated. The moon's mean diſtance is $60\frac{1}{7}$ ſemidiameters of the earth; and the mean inclination of her orbit is about 5°.8′. The nodes are retrograde at the rate of 19°.19′.45″ in a year of 365*d*.; and the apogee is progreſſive at the rate of 6′.41″.1‴ in a day.

ON THE SATELLITES OF JUPITER, SATURN AND THE GEORGIUM SIDUS.

72. Jupiter has four ſatellites revolving about it.

Their periodic times are 1*d*.18*h*.27′.33″; 3*d*.13*h*.13′.42″; 7*d*.3*h*.42′.33″ and 16*d*. 16*h*. 32′.8″. They revolve in orbits circular, or very nearly ſo, except the fourth which Dr. BRADLEY found to be elliptical. Their diſtances from the center of Jupiter in terms of his ſemidiameter are 5,965, 9,494, 15,141 and 26,63 according to Sir I. NEWTON. Their motions are ſubject to conſiderable irregularities from their mutual attractions. They ſuffer eclipſes like our moon.

73. Saturn has ſeven ſatellites revolving about it.

Their periodic times are 1*d*.8*h*.53′.8″,9; 1*d*.10*h*.37′.22″,9; 1*d*.21*h*.18′.27″; 2*d*.17*h*.41′.22″; 4*d*.12*h*.25′.12″; 15*d*.22*h*.41′.12″; 79*d*.7*h*.49′; the two former were determined by Dr. HERSCHEL, who diſcovered thoſe two ſatellites, and the other by Mr. POUND. The diſtance of the two firſt from the center of Saturn are 36′.79″ and 29′.67″ according to Dr. HERSCHEL, and of the others $43\frac{1}{2}$″; 56″; 1′.18″; 3′; 8′.$42\frac{1}{2}$″ according to M. CASSINI, the ſemidiameter of Saturn being 20″.

74. Saturn is alſo encompaſſed with a broad flat ring, whoſe greateſt apparent diameter : that of the planet :: 9 : 4.

This, which was ſuppoſed to be only one ring, Dr. HERSCHEL has diſcovered to be two, both lying exactly in the ſame plane. He has alſo diſcovered that it revolves about an axis perpendicular to its plane in 10*h*.32′.15″,4. Dr. HERSCHEL has alſo confirmed, what Sir I. NEWTON had before obſerved, that the 5th. ſatellite revolves about its axis; he makes the time of rotation to be 79*d*. 7*h*.47′.

75. The Georgium Sidus has two ſatellites revolving about it.

Theſe were diſcovered by Dr. HERSCHEL; he makes the times of their ſynodic revolutions to be 8*d*.17*h*.1′.19″,3 and 13*d*.11*h*.5′.1″,5; and their diſtances from the planet 33″,09 and 44″,23 at the mean diſtance of the planet. Their orbits are nearly perpendicular to the ecliptic.

ON

ON ECLIPSES.

76. The ſun is greater than the earth, and the earth is greater than the moon.

77. If tangents be drawn from the ſun to the *correſponding* ſides of an opaque body, they terminate the umbra; if they be drawn to the *oppoſite* ſides they terminate the penumbra.

Hence if the opaque body be a ſphere, the umbra and penumbra will be conical, and the latter will always be an increaſing fruſtrum of a cone; but as the earth and moon are leſs than the ſun, the former will be a cone which terminates. Hence a ſection of each perpendicular to the axis is a circle, and the penumbra includes the umbra.

The length of the earth's umbra, at the mean diſtance of the ſun, is about 216 ſemidiameters of the earth, and the length of the moon's umbra about 59.

In eclipſes of the moon the ſhadow is found to be a little greater than what this rule gives, owing to the atmoſphere of the earth. Mayer thinks you ought to add as many ſeconds as the ſemidiameter contains minutes.

78. An eclipſe of the ſun happens at the new moon, and an eclipſe of the moon at the full moon.

79. There is not an eclipſe of the ſun at every new moon, nor an eclipſe of the moon at every full moon, owing to the plane of the moon's orbit being inclined to the ecliptic.

80. To determine the limits of a lunar eclipſe.

The limits of a lunar eclipſe are theſe. At the full moon,

1. If the moon's latitude = the ſemidiameter of the earth's ſhadow + the ſemidiameter of the moon, there will be *no* eclipſe.

2. If the latitude be leſs than that ſum, but greater than their difference, there will be a *partial* eclipſe.

3. If the latitude be leſs than their difference, there will be a *total* eclipſe.

Cor. The diſtance of the center of the earth's ſhadow from the node, or the ſun's diſtance from the oppoſite node, at the firſt limit, is about $12^{\circ}.23'$; within which diſtance therefore the node muſt be from oppoſition that there may be an eclipſe of the moon.

What is here uſually called the moon's latitude is not ſtrictly ſo, it being

being a perpendicular to the moon's orbit and not to the ecliptic; it differs however but a very little from it, on account of the ſmall angle between them. The ſame quantity is called the latitude in the next propoſition.

81. To determine the limits of a ſolar eclipſe.

The limits of a ſolar eclipſe are theſe. At the new moon,

1. If the moon's latitude = the ſemidiameter of the earth + the ſemidiameter of the moon, there will be *no* eclipſe.
2. If the latitude be leſs than that ſum, but greater than their difference, there will be a *partial* eclipſe.
3. If the latitude be leſs than their difference, there will be a *total* eclipſe.

Cor. The diſtance of the ſun from the node at the firſt poſition is about $16°.35'$; within which diſtance therefore the earth muſt be from the node that there may be an eclipſe.

82. There are more eclipſes of the ſun than of the moon; but more eclipſes of the moon than of the ſun are ſeen at any given place.

If the nodes of the moon's orbit were fixed, there could not be more than ſix eclipſes in a year, nor leſs than two; but in conſequence of the retrograde motion of the moon's nodes, there may be ſeven in a year, but this does not affect the other limit.

TO FIND THE LONGITUDE.

83. The longitude may be found by an eclipſe of the moon, or of Jupiter's ſatellites.

For having the hour when they happen at Greenwich, find by prop. 16, 17. the hour when they happen at any other place, and the difference of the times converted into degrees at the rate of 15° for an hour gives the difference of longitudes; and the longitude of the place is eaſt or weſt from Greenwich, according as the hour at the place is before or after that at Greenwich. The eclipſes of Jupiter cannot be obſerved at ſea from the unſteadineſs of the ſhip.

84. The longitude may alſo be found by a watch which keeps true time.

For if ſet to the time at Greenwich, it will always ſhow the time at that place, which compared with the time found at any other place, by taking the ſun's or a ſtar's altitude, the difference gives the longitude as before.

85. The

85. The longitude may be found by taking the moon's diſtance from the ſun or a fixed ſtar.

Obſerve the diſtance, and alſo the two altitudes, and from thence find the true diſtance, and alſo the time from the ſun's or ſtar's altitude. The true diſtance of the moon from the ſun or ſtar is put down in the Nautical Almanack for every three hours at Greenwich; hence, by proportion, you may find the time at Greenwich when it is at any other diſtance. Find therefore the time at Greenwich when it is at the true diſtance deduced from obſervation, and the difference between that time and the time at the place of obſervation gives the difference of longitudes. See all the rules, with an example, in my *Practical Aſtronomy*, p. 51.

ON THE PROGRESSIVE MOTION OF LIGHT, AND THE ABERRATION OF THE FIXED STARS.

86. Light is not inſtantaneous but progreſſive.

This was diſcovered by M. REAUMUR, who obſerved that an eclipſe of Jupiter's ſatellites began about 16′ ſooner when Jupiter was in oppoſition than when in conjunction, which he determined could ariſe from no other cauſe than the progreſſive motion of light, which would have to move over the diameter of the earth's orbit more in the latter caſe than in the former, before the eclipſe would appear to begin.

87. From the progreſſive motion of light and of the earth in its orbit, the place of every fixed ſtar, meaſured by an inſtrument, is forwarder than its true place, reckoned according to the motion of the ſpectator.

88. The ſine of the angle of aberration : the ſine of the angle which the line joining the earth and the apparent place of the ſtar makes with the direction of the earth's motion :: the velocity of the earth : the velocity of light.

Cor. As the former angle is obſerved to be 20″ when the latter becomes a right angle, we have, the velocity of the earth : velocity of light :: ſin. 20″ : rad. :: 1 : 10314. Hence alſo this aberration is nothing when the ſpectator moves directly to or from a ſtar. The place meaſured by the inſtrument is called the *apparent* place.

89. Whilſt

89. Whilſt the earth makes one revolution in its orbit, the curve deſcribed by the apparent place of a ſtar upon a plane parallel to the ecliptic is a circle.

The true place of the ſtar in that circle divides that diameter, which, if projected upon the ecliptic, would be perpendicular to the major axis of the earth's orbit, in the ſame ratio as one of the foci divides that major axis.

90. Every ſtar *appears* to deſcribe an ellipſe, whoſe major axis : minor :: radius : ſine of the ſtar's latitude.

The major axis of this ellipſe for all ſtars is 40″, and is always parallel to the ecliptic; the minor axis is perpendicular to it. The path appears an ellipſe, becauſe it is the abovementioned circle ſeen obliquely.

91. If m and n be the ſine and coſine of the earth's diſtance from ſyzygies, v and w the ſine and coſine of the ſtar's latitude to radius unity; then $vm \times 20'' =$ the aberration in latitude, and $\frac{n}{w} \times 20'' =$ the aberration in longitude.

The exact agreement of this theory with obſervations is a very ſatiſfactory proof of the velocity of the earth, and that light is progreſſive. In 70 obſervations on γ *Draconis*, Dr. BRADLEY found but one (and that is noted very dubious on account of clouds) which differed more than 2″ from theory, and that did not differ 3″. The theory agreeing thus exactly with obſervation, without conſidering the annual parallax, proves that the annual parallax is inſenſible. Hence Dr. BRADLEY deduced the following concluſions. 1. That the light of all the fixed ſtars arrives at the earth with the ſame velocity; for the major axis of the ellipſe is the ſame in all the ſtars. 2. That unleſs their diſtances from us are all equal, which is very improbable, their lights are propagated uniformly to all diſtances. 3. That light comes from the ſun to the earth in 8′.7½″, and its velocity : that of the earth :: 10314 : 1. 4. That as the velocity of the ſtar light comes out about a mean of the ſeveral velocities found from the eclipſes of Jupiter's ſatellites, we may conclude that the velocity of reflected light is equal to that of direct light. 5. And as it is highly probable that the velocity of light from the ſun and fixed ſtars is the ſame, it follows that its velocity is not altered by reflection into the ſame medium.

ON THE PHYSICAL CAUSES OF THE PLANETS MOTIONS.

92. All the primary planets are urged by forces tending to the ſun.

For by obſervation they all deſcribe equal areas about the ſun in equal times, therefore by Sir I. NEWTON'S PRINCIPIA, prop. 2. ſect. 2. lib. 1. they are urged by forces tending to the ſun. This is called a *centripetal* force. The projectile force of each planet prevents them from deſcending to the ſun, and this is called a *centrifugal* force. By theſe two forces the planets are retained in their orbits.

93. All the primary planets deſcribe ellipſes about the ſun, which is in one of the foci.

For by obſervation the diſtance of the apſides of their orbits in reſpect to the ſun $=180°$, therefore by the PRINCIPIA, ſect. 9. prop. 45. lib. 1. the force varies inverſely as the ſquare of their diſtance from the ſun; hence by ſect. 3. prop. 13. cor. they deſcribe ſome conic ſection having the ſun in one of the foci, and that conic ſection muſt be an ellipſe, otherwiſe the orbit would not have two apſides. Hence the force with which the primary planets are urged towards the ſun varies inverſely as the ſquare of their diſtance from the ſun.

94. All the ſecondary planets are urged by forces tending to their reſpective primaries.

For they all deſcribe areas about their reſpective primaries proportional to the times, except ſome ſmall irregularities which may be accounted for from the action of the ſun, and from their actions upon each other; hence, as before, they are urged by forces tending to their reſpective primaries.

95. If the ſun did not diſturb the motion of the moon, it would revolve in an ellipſe about the earth in one of its foci.

The apſides of the moon's orbit have a ſmall progreſſive motion. Now it appears by computation, that this motion is juſt as much as the ſun would cauſe, if the moon revolved in an ellipſe about the earth in one of its foci; therefore without ſuch a diſturbing force the moon would ſo revolve. Hence the moon is urged towards the earth by a force which varies inverſely as the ſquare of the diſtance.

96. All the ſecondary planets are urged towards their reſpective primaries by forces which vary inverſely

verſely as the ſquares of their diſtances from the primary.

For they all revolve in circles, or very nearly ſo, and the periodic times are obſerved to be in a ſeſquiplicate ratio of their diſtances. Hence by the PRINCIPIA, lib. 1. ſect. 2. prop. 4. cor. 6. the force varies inverſely as the ſquare of the diſtance.

All the deviations of the moon's motion from its motion in an immoveable ellipſe, may be accounted for from the action of the ſun upon it.

The general principles of theſe irregularities are explained by Sir I. NEWTON in his PRINCIPIA, lib. 1. prop. 66. and its corrollaries. In lib. 3. he has computed the principal effects, and ſhown that they agree with obſervation. Since his time all the ſmaller effects have been determined, partly by theory and partly by obſervation, and tables conſtructed which will give the place of the moon to a very great degree of accuracy.

97. The earth is a ſpheroid whoſe polar diameter is leſs than the equatorial.

This ariſes from the centrifugal force of its parts from its rotation about its axis. According to Sir I. NEWTON, in lib. 3. prop. 19. the ratio of the diameters is as 229 : 230, which makes the equatorial exceed the polar diameter by about 34 miles.

98. The preceſſion of the equinoxes ariſes from the attraction of the ſun and moon upon the parts of the earth, exterior to that of a ſphere upon the polar diameter.

Sir I. NEWTON, in lib. 1. prop. 66. cor. 20. has given the principles upon which this motion of the earth may be accounted for; but in his inveſtigation of the quantity of the effect in lib. 3. prop. 39. he has fallen into an error. As however the earth is not homogeneous, nor the denſity of the moon accurately known, it is impoſſible to compute the true quantity with certainty.

99. The tides ariſe from the action of the ſun and moon.

The general principles of the tides, Sir I. NEWTON has explained in lib. 1. prop. 66. cor. 19; and in lib. 3. prop. 24. *de Syſt. Mundi,* he has ſhown that they will account for all the phænomena.

TABLE I.
The Refractions of the Heavenly Bodies in Altitude

App. Alt.		Refrac.		App. Alt.		Refrac.		App. Alt.		Refrac.	
D.	M.	M.	S.	D.	M.	M.	S.	D.	M.	M.	S.
0.	0	33.	0	3.	20	13.	34	9.	50	5.	20
0.	5	32.	10	3.	25	13.	20	10.	0	5.	15
0.	10	31.	22	3.	30	13.	6	10.	15	5.	7
0.	15	30.	35	3.	40	12.	40	10.	30	5.	0
0.	20	29.	50	3.	50	12.	15	10.	45	4.	53
0.	25	29.	6	4.	0	11.	51	11.	0	4.	47
0.	30	28.	22	4.	10	11.	29	11.	15	4.	40
0.	35	27.	41	4.	20	11.	8	11.	30	4.	34
0.	40	27.	0	4.	30	10.	48	11.	45	4.	29
0.	45	26.	20	4.	40	10.	29	12.	0	4.	23
0.	50	25.	42	4.	50	10.	11	12.	20	4.	16
0.	55	25.	5	5.	0	9.	54	12.	40	4.	9
1.	0	24.	29	5.	10	9.	38	13.	0	4.	3
1.	5	23.	54	5.	20	9.	23	13.	20	3.	57
1.	10	23.	20	5.	30	9.	8	13.	40	3.	51
1.	15	22.	47	5.	40	8.	54	14.	0	3.	45
1.	20	22.	15	5.	50	8.	41	14.	20	3.	40
1.	25	21.	44	6.	0	8.	28	14.	40	3.	35
1.	30	21.	15	6.	10	8.	15	15.	0	3.	30
1.	35	20.	46	6.	20	8.	3	15.	30	3.	24
1.	40	20.	18	6.	30	7.	51	16.	0	3.	17
1.	45	19.	51	6.	40	7.	40	16.	30	3.	10
1.	50	19.	25	6.	50	7.	30	17.	0	3.	4
1.	55	19.	0	7.	0	7.	20	17.	30	2.	59
2.	0	18.	35	7.	10	7.	11	18.	0	2.	54
2.	5	18.	11	7.	20	7.	2	18.	30	2.	49
2.	10	17.	48	7.	30	6.	53	19.	0	2.	44
2.	15	17.	26	7.	40	6.	45	19.	30	2.	39
2.	20	17.	4	7.	50	6.	37	20.	0	2.	35
2.	25	16.	44	8.	0	6.	29	20.	30	2.	31
2.	30	16.	24	8.	10	6.	22	21.	0	2.	27
2.	35	16.	4	8.	20	6.	15	21.	30	2.	24
2.	40	15.	45	8.	30	6.	8	22.	0	2.	20
2.	45	15.	27	8.	40	6.	1	23.	0	2.	14
2.	50	15.	9	8.	50	5.	55	24.	0	2.	7
2.	55	14.	52	9.	0	5.	48	25.	0	2.	2
3.	0	14.	36	9.	10	5.	42	26.	0	1.	56
3.	5	14.	20	9.	20	5.	36	27.	0	1.	51
3.	10	14.	4	9.	30	5.	31	28.	0	1.	47
3.	15	13.	49	9.	40	5.	25	29.	0	1.	42

Table I. continued.

App. Alt. D.	M.	Refrac. M.	S.
30.	0	1.	38
31.	0	1.	35
32.	0	1.	31
33.	0	1.	28
34.	0	1.	24
35.	0	1.	21
36.	0	1.	18
37.	0	1.	16
38.	0	1.	13
39.	0	1.	10
40.	0	1.	8
41.	0	1.	5
42.	0	1.	3
43.	0	1.	1
44.	0	0.	59
45.	0	0.	57
46.	0	0.	55
47.	0	0.	53
48.	0	0.	51
49.	0	0.	49
50.	0	0.	48
51.	0	0.	46
52.	0	0.	44
53.	0	0.	43
54.	0	0.	41
55.	0	0.	40
56.	0	0.	38
57.	0	0.	37
58.	0	0.	35
59.	0	0.	34
60.	0	0.	33
61.	0	0.	32
62.	0	0.	30
63.	0	0.	29
64.	0	0.	28
65.	0	0.	26
66.	0	0.	25
67.	0	0.	24
68.	0	0.	23
69.	0	0.	22
70.	0	0.	21
71.	0	0.	19
72.	0	0.	18
73.	0	0.	17
74.	0	0.	16
75.	0	0.	15
76.	0	0.	14
77.	0	0.	13
78.	0	0.	12
79.	0	0.	11
80.	0	0.	10
81.	0	0.	9
82.	0	0.	8
83.	0	0.	7
84.	0	0.	6
85.	0	0.	5
86.	0	0.	4
87.	0	0.	3
88.	0	0.	2
89.	0	0.	1

Table II.
Dec. Parts of an Hour.

1	,01666	1″	,00028
2	,03333	2	,00056
3	,04999	3	,00083
4	,06666	4	,00111
5	,08333	5	,00139
6	,09999	6	,00167
7	,11666	7	,00194
8	,13333	8	,00222
9	,14999	9	,00250
10	,16666	10	,00277
20	,33333	20	,00556
30	,49999	30	,00833
40	,66666	40	,01111
50	,83333	50	,01388

Table III.
For converting Time into Deg. Min. and Sec. at the Rate of 24 Hours for 360°.

Hou	Deg.	Min Sec.	Deg. Min.	Min. Sec.	Dec. o Sec	Sec.
1	15	1	0.	15	,1	1,5
2	30	2	0.	30	,2	3,0
3	45	3	0.	45	,3	4,5
4	60	4	1.	00	,4	6,0
5	75	5	1.	15	,5	7,5
6	90	6	1.	30	,6	9,0
7	105	7	1.	45	,7	10,5
8	120	8	2.	00	,8	12,0
9	135	9	2.	15	,9	13,5
10	150	10	2.	30		
11	165	20	5.	00		
12	180	30	7.	30		
16	240	40	10.	00		
20	300	50	12.	30		

TABLE IV.

For converting Deg. Min. and Sec. into Time, at the Rate of 360° for 24 Hours.

Deg. Min.	Hou. Min.	Min. Sec.	Deg. Min.	Hou. Min.	Min. Sec.	Sec.	Dec. of Sec.
1	0.	4	30	2.	00	1	,067
2	0.	8	40	2.	40	2	,133
3	0.	12	50	3.	20	3	,199
4	0.	16	60	4.	00	4	,266
5	0.	20	70	4.	40	5	,333
6	0.	24	80	5.	20	6	,399
7	0.	28	90	6.	00	7	,466
8	0.	32	100	6.	40	8	,533
9	0.	36	200	13.	20	9	,599
10	0.	40	300	20.	00	10	,666
20	1.	20					

TABLE V.

Length of circular Arcs to Rad. = 1.

1°	,0174533	1′	,0002909	1″	,00000485
2	,0349066	2	,0005818	2	,00000970
3	,0523599	3	,0008727	3	,00001454
4	,0698132	4	,0011636	4	,00001939
5	,0872665	5	,0014544	5	,00002424
6	,1047198	6	,0017453	6	,00002909
7	,1221730	7	,0020362	7	,00003394
8	,1396263	8	,0023271	8	,00003878
9	,1570796	9	,0026180	9	,00004363
10	,1745329	10	,0029089	10	,00004848

MAGNETISM.

1. THE earth contains a mineral ſubſtance which attracts iron, ſteel and all ferruginous ſubſtances; this is called a *natural magnet* or *loadſtone*.

The ſame ſubſtance has the power to communicate its properties to iron, ſteel and all ferruginous ſubſtances; the bodies to which theſe properties are communicated are called *artificial magnets*. Theſe magnets are alſo made without the natural magnet.

2. If a magnet be ſuſpended at its center of gravity upon a point, ſo that it may revolve in an horizontal plane, one end will be conſtantly turned towards the north pole of the earth and the other towards the ſouth pole. The end directed towards the north pole of the earth is called the *north pole* of the magnet, and the other end the *ſouth pole*. This is called a *compaſs*.

Every magnet has two poles of this kind. The *north pole* is not directed exactly towards the north pole of the earth; the preſent variation at London, is about $23\frac{3}{4}°$ towards the weſt, and is increaſing. In the year 1657, the magnet was directed exactly towards the north pole. Before that time it was directed towards the eaſt; but ever ſince that time the deviation has been increaſing towards the weſt. In different parts of the earth's ſurface the deviation is different. This deviation from the north is called the *variation* of the compaſs. The variation at the ſame place is obſerved to be different at different times of the year, and on different parts of the day, it being affected by heat and cold.

A great circle in the heavens paſſing through thoſe two points of the horizon to which the poles of a magnet are directed is called the *magnetic meridian*.

3. If

3. If two magnets be ſuſpended and placed near to each other, the *ſame* poles will *repel* and the *contrary* will *attract* each other.

The point between the two poles where the magnet has no attraction nor repulſion is called the *magnetic center.* This point is not always exactly in the middle between the two poles.

4. If two magnets be made to ſwim upon any fluid, the *contrary* poles will *attract* each other and bring them together; and the *ſame* poles will *repel* and ſeparate them.

5. If one end of a magnet be drawn along a needle, or any bar of iron or ſteel, ſeveral times in the ſame direction, the needle or bar will become magnetic; and that extremity of the needle or bar which the magnet touched laſt, acquires a polarity contrary to that of the end of the magnet which was applied.

6. If the magnet be drawn in a contrary direction, it will take away the magnetiſm which it before gave.

7. A bar of iron or ſteel not magnetic attracts a magnet as much as the magnet attracts the bar.

8. The magnetic power will be increaſed in two magnets by letting them remain with their oppoſite poles together.

Magnetic bars ſhould therefore be always left with the oppoſite poles laid againſt each other, or by connecting their oppoſite poles by a bar of iron; if the like poles be laid together, they will diminiſh or deſtroy each other's magnetiſm.

The magnetic power will be increaſed in a magnet, by letting a piece of iron remain attached to one or both of its poles. A ſingle magnet ſhould therefore be always thus left.

9. A bar of ſoft ſteel acquires magnetiſm faſter and loſes it faſter than an hard bar.

10. To find whether a bar of iron or ſteel be magnetic, apply one end to the needle, and if it attract both ends of the needle it is not magnetic; but if it attract one end and repel the other it is magnetic.

11. If a magnetic bar be broken into any two parts, each part becomes a complete magnet having two poles, the ends of each next to where it was broken, acquiring a polarity contrary to the other end.

12. A magnet will attract a piece of iron or ſteel, although any other body ſhould interpoſe, unleſs that body be red hot.

13. The attraction between two magnets begin at a greater diſtance than between a magnet and a piece of iron; but when in contact the magnet attracts the magnet with leſs force than it attracts the iron.

It appears from hence, that the attraction varies faſter in the latter caſe than in the former.

14. If the contrary poles of a ſtrong and weak magnet be placed together, it often happens that the weaker will have its poles changed, depending upon their different ſtrengths.

Without attending to this it might appear that the ſame poles attract each other. But this happens for the ſame reaſon that a magnet will give magnetiſm to any piece of iron; it firſt deſtroys the ſame magnetiſm and then gives it a contrary one.

15. If a bar or needle be laid in the magnetic meri-

meridian and an electrical ſtroke be ſent through it, that end towards the north acquires a north polarity, and the other the ſouth polarity.

16. If a bar of ſoft iron be kept vertical for ſome time in theſe parts of the earth, it becomes magnetic, and the lower end will acquire a north polarity; but in the ſouthern parts of the earth it acquires a ſouth polarity: if you invert the bar its polarity will be inſtantly reverſed.

17. A long bar of iron or ſteel, or a long piece of wire, ſuppoſe 3 or 4 feet, may have ſeveral poles, the north and ſouth following one another with a magnetic center between each.

18. An iron bar which in one poſition will attract one end of a magnet, will when held in another poſition repel it.

19. If a magnet be ſuſpended by an horizontal axis at its center of gravity, ſo that it may vibrate in a vertical circle, the north pole will here be depreſſed and the ſouth pole elevated; in the ſouthern parts of the earth, the contrary takes place; and the dip alters gradually from one hemiſphere of the earth to the other.

This inſtrument is called the *dipping* needle; and, like the poſition when moving in an horizontal plane, the dip is ſubject to a variation. The dip at London, at this time is about $72\frac{1}{2}°$, and from the moſt accurate obſervations on that dipping needle belonging to the Royal Society, it appears to diminiſh about $15'$ in 4 years. In going from the north to the ſouth, the dip does not alter regularly. As it is extremely difficult to balance the needle accurately, the poles of the needle are generally reverſed by a magnet, ſo that its two ends may dip alternately, and the mean of the two is taken.

The phænomena of the *compaſs* and *dipping needle*, and of the magnetiſm acquired by an iron bar in a vertical poſition, leave no room to

doubt

doubt but that the cauſe exiſts in the earth. Dr. HALLEY ſuppoſed that the earth has within it a large magnetic globe, not fixed within to the external parts, having four magnetic poles, two fixed and two moveable, which will account for all the phænomena of the compaſs and dipping needle. This would make the variation ſubject to a conſtant law, whereas we find cauſual changes which cannot be accounted for upon this hypotheſis. This the Doctor ſuppoſes may ariſe from an unequal and irregular diſtribution of the magnetical matter. The irregular diſtribution alſo of ferruginous matter in the ſhell may alſo cauſe ſome irregularities. Mr. CAVALLO's opinion is, that the magnetiſm of the earth ariſes from the magnetic ſubſtances therein contained, and that the magnetic poles may be conſidered as the centers of the polarities of all the particular aggregates of the magnetic ſubſtances; and as theſe ſubſtances are ſubject to change, the poles will change. Perhaps it may not be eaſy to conceive how theſe ſubſtances can have changed ſo materially, as to have cauſed ſo great a variation in the poles, the poſition of the compaſs having changed from the eaſt towards the weſt about 33° in 200 years. Alſo the gradual, though not exactly regular, change of variation ſhows, I think, that it cannot depend upon the accidental changes which may take place in the matter of the earth. Dr. HALLEY firſt laid down charts of the variation of the compaſs, drawing curve lines through all thoſe places where the variation is the ſame. Theſe curve lines never cut one another. If a dipping needle be carried from one end of a magnetic bar to the other, when it ſtands over the ſouth pole the north end of the needle will be directed perpendicularly to it; as the needle is moved, the dip will grow leſs, and when it comes to the magnetic center it will ſtand parallel to the bar; afterwards the ſouth end will dip, and the needle will ſtand perpendicular to the bar when it is directly over the north pole. A bar thus uſed is called a *terella*, or little earth, the phænomena being ſimilar to thoſe of carrying a dipping needle from the north to the ſouth. According to Dr. HALLEY's Hypotheſis, the pole of the magnet within the earth, which makes our needle dip here has a ſouth polarity, as it attracts that which we call the north pole of the needle; and the contrary pole muſt have a north polarity. The *aurora borealis* appears to have an affect upon the needle, as it has often been obſerved to be diſturbed when that phænomenon has appeared very ſtrong.

ELEC-

ELECTRICITY.

1. If a glaſs tube be rubbed with ſilk, or if a globe or cylinder be turned about its axis and rub againſt a cuſhion covered with ſilk, ſparks and flaſhes of fire will dart from them. This is called electricity; and the luminous matter is called the electric fluid.

The excitation will be ſtronger if upon the ſilk you put ſome amalgam, uſually made with five parts of quickſilver and one of zinc.

The bodies which you can thus excite are called *electrics*; thoſe which you cannot excite are called *non-electrics*. Thoſe bodies which being applied to an excited electric receive and tranſmit the fluid are called *conductors*; thoſe which will not tranſmit the fluid are called *non-conductors*. Every *electric* is a *non-conductor*, and every *non-electric* is a *conductor*. The electrics, or non-conductors, are glaſs, ſealing-wax, roſins, amber, ſulphur, baked wood, ſilk, &c. &c. The non-electrics, or conductors, are metals, ores, quickſilver, all fluids except air and oils, moſt ſaline ſubſtances, ſtony ſubſtances, green wood, &c. &c. Amongſt theſe, metals are the beſt conductors. A body is ſaid to be *inſulated* when it is ſupported by non-conductors only. The electric fluid is generally excited by a cylinder and cuſhion, and againſt the cylinder an inſulated conductor, called the *prime conductor*, is placed to receive and contain the fluid as it is excited. The cuſhion is inſulated, but that inſulation may be taken away by hanging a chain from it to the ground.

2. If an electric tube be excited, it will firſt attract and then repel light bodies, as ſmall pieces of paper, thread, metal, &c. &c. The conductor will alſo do the ſame.

3. If two light pith or cork balls be hung together from a non-conductor, or be insulated in any manner, when brought near the prime conductor they will repel each other.

4. If an insulated conductor be connected with the cushion, and there be two pith or cork balls insulated and electrified by it, these balls repel each other.

5. If one insulated ball be electrified by the prime conductor, and another by the conductor on the cushion, when brought together they will attract each other.

6. If one ball be electrified by a smooth excited glass tube, and the other by an excited cylinder of sealing-wax or rosin, they will attract each other.

7. If one ball be electrified by a smooth and the other by a rough excited glass tube, they will attract each other.

8. If a ball be electrified by any excited body, it will attract a body not electrified.

It appears from these experiments that there are two different electric powers; that from the prime conductor, or smooth glass tube, is called *positive*, and sometimes *vitreous* electricity; that from the cushion, sealing-wax, rosin or rough glass tube is called *negative*, and sometimes *resinous* electricity.

Hence the following properties of electric attraction and repulsion.

1. If two bodies be electrified both positively or both negatively, they repel each other.

2. If one be electrified positively and the other negatively, they will attract each other.

3. A body electrified either positively or negatively will attract a body not electrified.

HYPO-

HYPOTHESES.

Refpecting the theory of electricity there are two different Hypothefes, one that there is only one fluid, and the other that there are two. Dr. FRANKLIN's Hypothefis is, that there is only one fluid, and this theory depends on the following principles. 1. That all terreftrial bodies are full of the electric fluid. 2. That the electric fluid violently repels itfelf and attracts all other matter. 3. By exciting an electric the equilibrium of the electric fluid contained in it is deftroyed, and one part contains more than its natural quantity, and the other lefs. 4. Conducting bodies connected with that part which contains more electric fluid than its natural quantity, receive it, and are charged with more than their natural quantity; this is called *pofitive* electricity; if they be connected with that part which has lefs than its natural quantity, they part with fome of their own fluid, and contain lefs than their natural quantity; this is called *negative* electricity. 5. When one body pofitively and another negatively electrified are connected by any conducting fubftance, the fluid in the body which is pofitively electrified rufhes to that which is negatively electrified, and the equilibrium is reftored. Thefe are the principles of *pofitive* and *negative* electricity. The other Hypothefis is, that there are two diftinct fluids; this was firft fuggefted by Mr. DU FAYE, upon his difcovery of the different properties of excited glafs, and excited rofins, fealing-wax, fulphur, &c. The following are the principles of this theory. 1. That the two powers arife from two different fluids which exift together in all bodies. 2. That thefe fluids are feparated in non-electrics, by the excitation of electrics, and from thence they become evident to the fenfes, they deftroying each others effects when united. 3. When feparated they rufh together again with great violence in confequence of their ftrong mutual attraction, as foon as they are connected by any conducting fubftance. Thefe are the principles of *vitreous* and *refinous* electricity. This is the theory of Mr. EELES from the hint of two fluids by Mr. DU FAYE.

9. The electric fluid is received from the earth.

For if the rubber be infulated no electricity can be produced.

A jar or phial coated on the infide and outfide with tinfoil, except about 2 inches on the top, is called the *Leyden* phial or jar, the ufes of which in electricity having been difcovered at Leyden, by Mr. MUSCHENBROEK.

10. If a brafs wire, with a ball on the top, be put into the Leyden phial, and the ball be applied to the prime conductor when electrified, and the outfide be connected with the earth by conductors, the infide will be charged with the pofitive, and the

the outſide with the negative electricity; and if a communication be formed by a conducting ſubſtance between the inſide and the outſide, an exploſion will be heared, and the phial will be diſcharged.

The communication may be made by a number of perſons taking hold of each others hand, when each perſon will receive a ſhock.

If this be explained by the firſt hypotheſis, the inſide of the jar is ſaid to have more than its natural quantity of the electric fluid, and the outſide as much leſs than its natural quantity, one always gaining exactly as much as the other loſes. If it be explained by the ſecond hypotheſis, the inſide of the jar is ſaid to have received a certain quantity of the vitreous fluid, and the outſide as much of the reſinous.

11. If the ball of the phial be applied to the conductor on the cuſhion, the inſide will be charged with the negative and the outſide with the poſitive electricity.

As it is neceſſary for many experiments to collect a large quantity of this fluid, a great many phials are placed together, having their outſides and inſides reſpectively connected, in conſequence of which they may all be diſcharged together. This is called a battery.

12. If the outſide of an inſulated phial be connected with the prime conductor, and the inſide of another phial with the inſide of that, the outſide of the latter having a communication with the earth, both will be equally charged.

In this caſe the inſide of the ſecond phial has received as much fluid from the inſide of the firſt, as the outſide thereof received from the machine.

13. If a phial be inſulated, it cannot be charged by applying the knob from the wire within to the conductor.

14. Place an inſulated phial ſo that its knob may be about half an inch from the conductor, and whilſt the cylinder is turning, hold a braſs knob

knob near the coating of the jar, and it will receive a ſpark from the coating for every one that paſſes from the conductor to the knob.

Theſe two laſt facts prove, according to Dr. FRANKLIN's Hypotheſis, that the inſide can receive no fluid, unleſs an equal quantity goes off from the outſide.

15. Place the knob of an inſulated bottle in contact with the prime conductor, and connect the outſide coating with the cuſhion, and the bottle will be charged with its own electricity, the fire going from the outſide round by the cuſhion into the inſide.

16. Hang a ſmall linen thread near the coating of an electrified phial, and touch the wire from the inſide, and at every touch the thread will be attracted by the coating.

This Dr. FRANKLIN gave as a proof of his Hypotheſis; for as the fire was taken from the inſide, the outſide drew in an equal quantity by the thread.

17. When a phial is charged, the fluid reſides in the glaſs, and not in the coating.

18. If a phial be inſulated, and its knob connected to the prime conductor, and the machine be put in motion, a certain quantity of electric fluid will be added to the inſide; for if you touch the outſide, a quantity nearly equal to that thrown in comes from it.

This experiment is uſed by Dr. GREY againſt Dr. FRANKLIN's Hypotheſis, as here was a certain quantity of fluid thrown into the inſide without any coming from the outſide.

19. If a quire of paper be ſuſpended ſo that it may vibrate freely, and an electrical charge be ſent through

through it, it will give it no motion, but the leaves will be protruded both ways from the middle.

Upon ſuppoſition that there is only one fluid, it has appeared extraordinary that it ſhould not give the paper any motion. But, as Mr. Atwood obſerves, ſuch a velocity to the fluid may be aſſigned as ſhall give a ſmaller angular velocity to the paper than any that can be aſſigned, and we know no limit to the velocity of the electric fluid. But the oppoſite directions in which the leaves are protruded, tends to ſtrengthen the opinion of two fluids coming in an oppoſite direction.

20. Inſulate a charged bottle, and let a cork ball, ſuſpended by ſilk, hang againſt the outſide; touch the knob, and the ball flies off electrified with the reſinous power.

In this caſe, ſays Mr. Eeles, can any Franklinian ſuppoſe, that it is the return of the poſitive power to the outſide of the glaſs, that electrifies the ball negatively? He further obſerves, that it is abſurd to ſuppoſe that bodies negatively electrified are deprived of their natural ſhare of electricity; it being a contradiction to imagine, that bodies will repel each other the more, the more they are diveſted of the power of repulſion.

21. If a braſs wire and ball be fixed to the end of the conductor, and another ball be preſented to it, a crooked ſpark will paſs from one to the other, when the machine is highly charged.

The largeſt ſparks proceed from that end of the conductor which is furtheſt from the cylinder, and is nearly in proportion to the ſurface of the conductor.

22. If you preſent only a pointed wire, the electric fluid will paſs to it with a hiſſing noiſe and in a continual ſtream. The ſame follows, if you preſent a ball to a point from the conductor. If both be points, the electricity is more readily diſcharged.

Hence pointed conductors are put upon buildings in order to draw off the electric matter gradually from the clouds, by which they may prevent an accumulation of the fluid in the clouds, and thereby a ſtroke of lightning may be avoided. If a ſtroke ſhould happen to fall by the ſudden coming of a cloud over charged, the conductor may carry it

off

off without its hurting the building. The conductors ſhould be elevated two or three yards above the building, and the more there are put upon different parts of the building, the greater will be the ſafety.

23. The electric fluid appears a diverging ſtream from a point electrified poſitively, and like a ſmall ſtar from a point negatively electrified.

It is inferred from hence, that the fluid goes from the point electrified poſitively, as a fluid diverging from a point will neceſſarily put on the appearance obſerved from the point electrified poſitively; and that it goes to the point electrified negatively, for as the point is ſurrounded with the fluid, it muſt break upon it all round from the ſame diſtance, and therefore put on the appearance of a ſtar. This experiment ſeems to be in favour of only one fluid; for if there were two fluids paſſing in oppoſite directions, it would not be eaſy perhaps to account for the different appearances in the two points.

24. If the diſcharge of a phial be made through the flame of a candle, the flame will be driven towards that part which is electrified negatively.

25. If the diſcharge be made through a pith ball, it will be driven towards that part which is electrified negatively.

Theſe two experiments favour the doctrine of a ſingle fluid.

26. Preſent a thin exhauſted flaſk to the conductor, the flaſk will be illuminated, and will have the appearance of the aurora borealis.

27. Gunpowder and ſpirits may be fired by the electric fluid.

28. The electric fluid is communicated through the body of the conductor, and not upon its ſurface.

29. The electricity which is communicated to a body lies on its ſurface.

For the quantity which a body receives is obſerved to be in proportion to its ſurface, and not to its ſolid content.

30. Smoke,

30. Smoke, and the vapour of hot water, will conduct the electric fluid.

31. A pane of glafs may be electrified on both fides with the fame power.

32. Cover two large boards with tinfoil, place them parallel, and connect one infulated with the prime conductor, and the other with the ground. Turn the machine, and the former will be electrified pofitively and the latter negatively; and the air between them keep the two powers afunder. Thus a plate of air is charged.

33. If there be two different conductors to make a difcharge, the difcharge will always be made through that which conducts beft, every thing elfe being the fame.

34. If the conductors be the fame, but one fhorter than the other, the difcharge will be made through the fhorteft.

35. If two electric plates be charged, and a communication be formed between the vitreous fide of one and the refinous of the other, no difcharge will follow, unlefs a communication be formed between the other two fides at the fame time.

Upon this Mr. ATWOOD has made the following obfervations. The natural electricity in the atmofphere is frequently difcharged in this manner: Two clouds being electrified with oppofite powers, the furfaces of the earth immediately under them are likewife electrified with powers contrary to thofe in the clouds above them; and the moifture of the earth forming a communication between the two contiguous charged furfaces, whenever the two clouds meet, there will follow a difcharge, both of the clouds and furfaces on the earth oppofed to them. If the earth fhould be dry, and confequently afford a refiftance to the union of

of the two electricities accumulated on or under its ſurface, there will follow an exploſion in the earth as well as in the atmoſphere, which will produce concuſſions and other phænomena which have frequently been obſerved to happen in dry ſeaſons, particularly in thoſe climates which are the moſt liable to ſtorms of thunder and lightning.

36. If a kite be raiſed in the air with a proper ſtring, the ſtring is obſerved to conduct a quantity of the electric fluid from the air.

Mr. Cavallo deduced the following circumſtances from his experiments.

1. The air is always electrified poſitively, and more in froſty than in warm weather.
2. The preſence of the clouds generally leſſen the electricity; ſometimes it has no effect, and it very ſeldom increaſes it a little.
3. When it rains the electricity is generally negative, and very ſeldom poſitive.
4. The aurora borealis ſeems not to effect the electricity.

Dr. Franklin has obſerved that the clouds are ſometimes negatively electrified.

Mr. Achard obſerved the electricity of the atmoſphere with a pair of pith balls attached to a reſinous rod placed above the roof of an houſe. This electrometer, from its ſimplicity, is preferable to any other for diſcovering merely when any electricity exiſts in the atmoſphere.

ON THE ANALOGY AND DIFFERENCE BETWEEN MAGNETISM AND ELECTRICITY.

The power of electricity is of two ſorts, poſitive and negative; bodies poſſeſſed of the ſame ſort of electricity repel each other, and thoſe poſſeſſed of different ſorts attract each other. In magnetiſm, every magnet has two poles, one of which always ſtands towards the north, and the other towards the ſouth; the ſame poles repel each other, and the contrary poles attract each other.

In electricity, when a body in its natural ſtate is brought near to one electrified, it acquires a contrary electricity and becomes attracted by it. In magnetiſm, when a ferruginous ſubſtance is brought near to one pole of a magnet, it acquires a contrary polarity, and becomes attracted by it.

One ſort of electricity cannot be produced by itſelf. In like manner, no body can have only one magnetic pole.

The electric virtue may be retained by *electrics*, but it eaſily pervades *non-electrics*. The magnetic virtue is retained by ferruginous bodies, but it eaſily pervades other bodies.

On the contrary, the magnetic power differs from the electric, in that it does not affect the fenfes with light, fmell, tafte or noife, as the electric does.

Magnets attract only iron, whereas the electric power attracts bodies of every fort.

The electric virtue refides on the furface of electrified bodies, whereas the magnetic is internal.

A magnet lofes nothing of its power by magnetizing other bodies; but an electrified body lofes part of its electricity by electrifying other bodies.

Mr. CAVALLO thinks thefe are the principal points of analogy and difference between magnetifm and electricity.

ADDENDA.

In *mechanics*, to prop. 17. add this corollary.

Hence the motion in any given direction is not altered by composition.

To the ſecond obſervation to prop. 21. add,

Hence if the back and two ſides be B, S, S' reſpectively, then P, R and R' will be as $\frac{B}{p}$, $\frac{S}{r}$ and $\frac{S'}{r'}$.

In *optics*, prop. 27. cor. 1. and prop. 57. obſer. 1. by the "apparent magnitudes of the image and object" is underſtood, the angles which they ſubtend at the eye, and not the apparent magnitudes according to the judgement which the mind forms of them.

CORRIGENDA.

Pag. 13. l. laſt but 2, for *three-fourths*, read *three-fifths*. Pag. 15. l. 30. for *will*, read *may*. Pag. 25. l. 7. for *FVv*, read *FTv*. Pag. 136. l. laſt, for 68, read 67.

The purchaſers of my PRACTICAL ASTRONOMY, are requeſted to make the following corrections.

Pag. 2. l. laſt, for $\frac{1}{500}$, read $\frac{1}{600}$. Pag. 4. l. 20, for $\frac{1}{500}$, read $\frac{1}{600}$. Pag. 40. l. 17. for *latter*, read *former*; and l. 18. for *former*, read *latter*. Pag. 43. l. laſt but 1, for *at an alt.* 44°.22′, read *in alt. an azim.* 44°.22′; and l. laſt, for ,2334 *of a degree* = ,9336′, read 2′, 334 = 9″ ,336 *in time*. Pag. 47. l. 27. for *below*, read *above*. Pag. 64. l. 24. for 4″, read 4′. Pag. 97. l. 7. for *a*, read *c*. Pag. 133. l. 19. for *images*, read *limbs*. Pag. 170. l. 20. for 28, read 4. In the *errata*, for Dr. USHER, Profeſſor of Aſtronomy, at Dublin, read Mr. RAMSDEN.

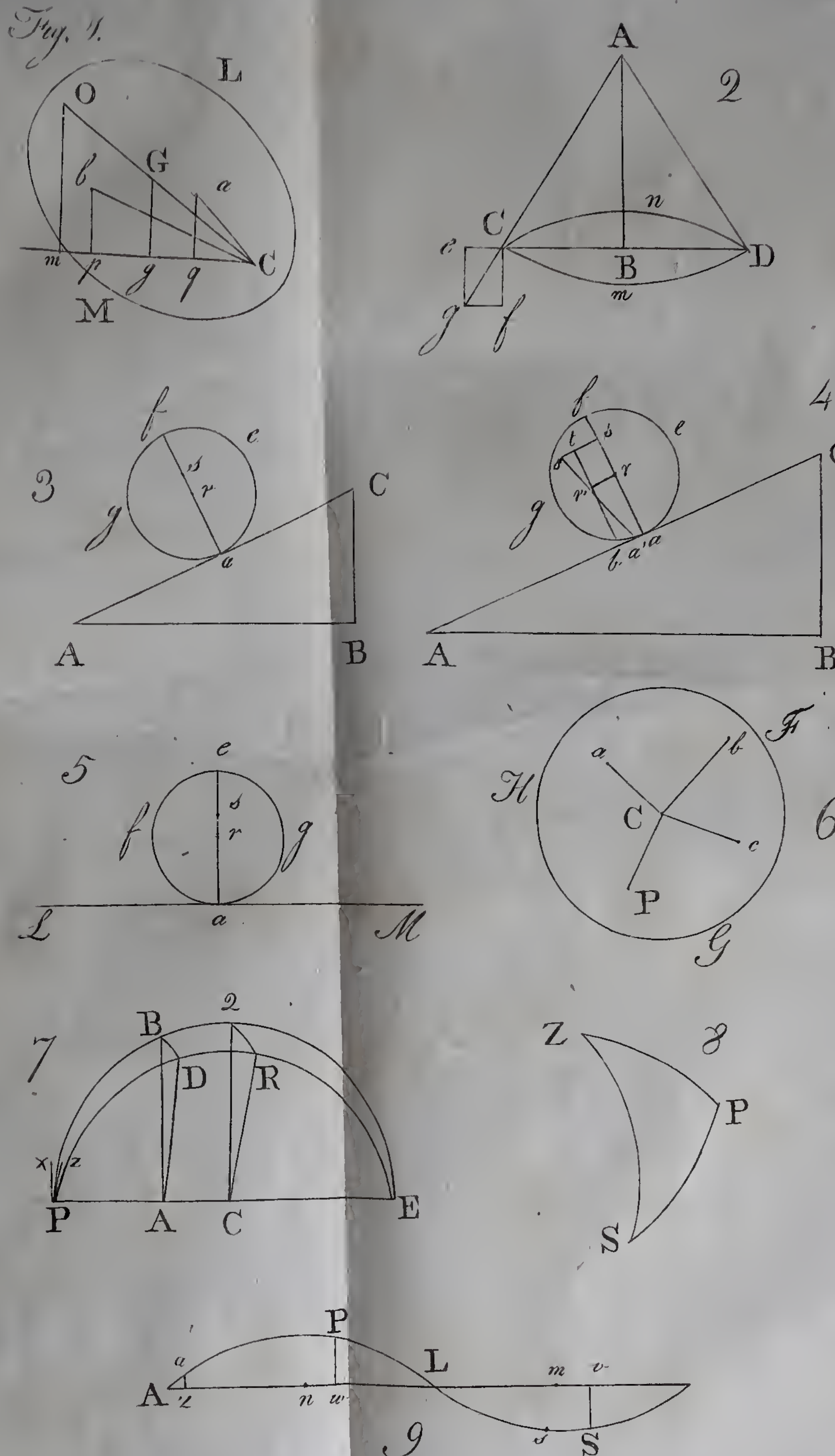

Fig. 1.
O L G b a m p g q C M
2
A n C e D B g f m
3
f e s r g C a A B
4
f e t s r g C b a a A B
5
e s r f g L a M
6
F H a b C c P G
7
B 2 D R x z P A C E
8
Z P S
9
P a L m v A z n w s S